Republic of Iraq  
Ministry of Education  
General Directorate of Curricula

Part Two

Series of Maths Books for Intermediate Stage

Mathematics

First Intermediate

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This series (Maths for Intermediate stage) has been edited by a special team of specialists in Ministry of Education/ General Directorate of curricula with participation of specialists from universities professors in Ministry of higher Education according to international standards to achieve the goals of designing modern syallabus which helps the students to be:
Successful learners long life
Self-stem individuals
Iraqi citizens feeling proud

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INTRODUCTION

The Maths subject is considered one of the basic courses that helps students to acquire educational abilities to develop his thinking and solving problems and it helping to deal with difficult situations in his life.

As a starting point of attention by the Ministry of Education represented by the General Directorate of curricula to develop the curricula in general and specially of Maths in order to go along with the technological and scientific development in different fields of life. A plan has set up to edit the series of Maths books for the three stages. Primary stage has been achieved and the work started to continue the series by editing the books of intermediate stage.

The series of new Iraqi Maths Books as a part of General frame work of curricula that reinforces the basic values as Iraqi identity, forgiveness, respecting different opinions, social justice and offering equal chance for creativity and it also reinforces abilities of thinking and learning, self-efficiency, action and citizenship efficiency. The series of Iraqi Maths books has been built on student-centered learning according to international standards.

The series of Iraqi maths books for intermediate stage has been built on six items: learn, make sure of your understanding, solve the exercises, solve environmental problems, think and write. The Maths book for first intermediate stage contains four basic fields: integer number operations, Algebra, geometry and measurement, statistics and probabilities for each field. The books consists of two parts. Part one for the first course which consists of four chapters, each chapter has its own exercises. Part two for the second course which consists of three chapters and each chapter has its own exercise.

The maths books have distinguished by presenting material in modern styles that attract and help the student to be active through presenting drills, exercises and environmental problems in addition there are extra exercises at the end of the book that are different from the exercises and drills in the lessons because they are objective so the student can answer through multiple choices and that prepare the student to participate the international competitions.

This book is an expansion for the series of developed Maths books for primary stage and it is also considered as support for the developed syllabus in maths and it also has a teachers book so we hope in applying them, the student will gain scientific and practical skills and develop their interest to study Maths.

We hope God help us to serve our country and our sons

Authors
Shanasheel: is a window in the wall covered with a frame that is composed of an overlap of a group of cylindrical small pieces (of an circular section) in the form of chains separated by specific distances and organized in a professional geometric way.
Pretest

1. Determine the points on the squares web and define the resulting shape.
   1) A (3,5), B (5,5), C (4,7)
   2) A (5,5), B (6,7), C (8,7), D (9,5)
   3) A (1,1), B (1,4), C (3,1), D (3,4)
   4) A (1,2), B (5,4), C (5,2), D (1,4)

2. Define some of the simple plane shapes from the following component shapes.
   ![Image 1](image1.png)
   ![Image 2](image2.png)

3. Plot the points on the coordinate plane then figure out the resulting shape for each of the followings.
   5) A (1,2), B (1,8), C (3,5)
   6) A (1,1), B (1,4), C (2,3), D (2,5)
   7) A (-1,-1), B (-2,-3), C (-6,-1), D (-4,-3)
   8) A (1,-1), B (4,-1), C (1,-4), D (4,-4)

4. Find the value of algebraic statement of each the followings by using the given variable.
   11) 2(n-3)+5, n=4
   12) x^2+2x+2, x=-1
   13) \( \frac{x^2}{2} + 3x + 7 \), x=4
   14) \( \frac{2x}{6} + 8 \), x=-3

5. Draw on the net of the square of the following three projection.
   ![Image 3](image3.png)

6. Determine the shape whose projections are given as shown in the followings.
   ![Image 4](image4.png)

7. Determine a model of a peripheral alignment using a pattern of squares and isosceles triangles.
In the picture on the side, there are traffic signs with different shapes including regular and irregular polygons. Classify the convex and concave polygons.

**Vocabulary:**
- *Interior angles*
- *Exterior angles*
- *Central angles*
- *Convex polygon*
- *Concave angles*

**Idea of the lesson:**
*describing the polygons, determining them and determining the interior, exterior and central angles and measuring them.

**Learn**

**[ 5-1-1 ] Convex and Concave Polygons**

You have previously learned the definition of the polygon and you got to know the regular polygon and the irregular one. In this lesson you will be introduced to the convex polygon and the concave polygon and you will learn to measure the interior, exterior and central angles of polygons.

*The polygon is called concave if at least one of its diagonal contained points located outside the polygon.*

*A polygon is convex if the points found in each diagonal were inside the polygon.*

Diagonal of the polygon: is each straight line connecting two non-adjacent vertices in the polygon and it is not a side in it.

**Example (1)**

Introduce the shapes in LEARN section and diagonal whether they are concave or convex polygons.

A convex quadrilateral polygon because all points of its diagonals are inside the polygon.

A concave hexagon polygon because it contains diagonal out of the polygon (hexagon)

A convex octagon polygon because the points of its diagonals are all inside the polygon (octagon).
**Interior angle:** it is the angle confined between any two adjacent sides in the polygon.

sum of interior angles in the polygon $180^\circ \times (n-2)$
where the number of sides is equal to $n$.

**Exterior angle:** it is the angle confined between any sides in the polygon and the extension of the adjacent side to it.

sum of exterior angle of the regular polygon is $360^\circ$.

**Central angle:** it is the angle whose vertex is the center of the regular polygon and its opposite side is one of the sides of the regular polygon.

Central angle of the regular polygon is equal to $\frac{360^\circ}{n}$.

---

**Example (2)** Find the Measure of sum of the interior angles in a regular hexagon.

Write the rule: $180^\circ \times (n-2)$

The number of sides in a regular hexagon is 6.

So substitute 6 for $n$: $180^\circ \times (6-2)$

So the Measure of sum of the interior angles in a hexagon is: $720^\circ$.

**Example (3)** Find the central angle of the polygon in example 2

Write the rule $\frac{360^\circ}{n}$

The number of sides in a hexagon is 6.

So substitute 6 for $n$.

So the Measure of the central angle in a regular hexagon is $\frac{360^\circ}{6} = 60^\circ$.

**Example (4)** Find the measure of each exterior angle in a regular octagon.

sum of exterior angle of the regular polygon is $360^\circ$.

There are 8 congruent interior angles so there are 8 congruent exterior angles.

The measure of each exterior angle in an octagon is $\frac{360^\circ}{8} = 45^\circ$. 
1. Complete the following table.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Polygon</th>
<th>Sum of interior angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td>180°</td>
</tr>
<tr>
<td>4</td>
<td>pentagon</td>
<td></td>
</tr>
</tbody>
</table>

Which of the following polygon is concave and which is convex?

[Descriptions of polygons]

Find the measurement of each interior angle in the following polygons:

- Heptagon (7 sides, 7 vertices)
- Octagon (8 sides, 8 vertices)
- Nonagon (9 sides, 9 vertices)

Which polygon has a total interior angles of?

- 540°
- 1260°
- 16260°

What is the measurement of the central angle of the regular polygon in the following?

- Pentagon
- Octagon
- Decagon
Which of the following polygon is concave and which is convex?

17

18

19

20

Find the measurement of each exterior angle of the following polygons

21

22

Solve the problems

23 Mosaic: the picture of mosaic represents the ceiling of one of the archaeological palaces, name the polygons found in it and determine whether they are concave or convex?

24 Call's Bee: a bee cell represents a hexagon(hexagon) what is the measurement of its exterior angle?

25 Chemistry: what polygon forms the molecule of hydrogen with oxygen in the picture?

what is the measurement of sum of its interior angle?

Think

26 Open problem: give examples of two polygons (triplex, pentagon) that represent things in life and find the measurement of their interior, exterior angles.

27 Challenge: which regular polygon has a central angle of 90°?

Write

Explain the relationship between the number of a regular polygon’s sides and the measurement of each interior angle it contains.
Solid Shapes and Composite Solid Shapes

Idea of the lesson:
Distinguishing the Polyhedron forms (solid shapes) and the composite Polyhedron forms and determining the number of faces, vertices and edges of each Polyhedron form (solid shape)

Vocabulary:
* Face
* Edge
* Vertix
* Prism
* Base

The Iraqi museum is one of the biggest and oldest museum in Iraq. it was establish in 1923. It contains monumental groups chronicling Mesopotamia. The entrance of the Iraqi museum is represented by polyphonic forms (solids). The gate is composed of a rectangular parallelepiped.

Properties of Polyhedron Formats

You have previously studied the polyhedron shapes and some of their properties and in this lesson you will learn more about them.
Polyhedron shapes: (three dimensional solids) are shapes of length, width and height. The following words are used to describe them in this lesson: edges, faces, vertices as shown in the example.

Example (1) What are the properties of solids ?

<table>
<thead>
<tr>
<th>Shape</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyramid</td>
<td>It has at least three triangle faces, one base in the form of a polygon. The shape of the base determines the name of the pyramid.</td>
</tr>
<tr>
<td>Cone</td>
<td>It has got only one base. The base is circular. It has one verticx.</td>
</tr>
<tr>
<td>Cylinder</td>
<td>It has only two circular bases. It has no vertices nor edges.</td>
</tr>
<tr>
<td>Sphere</td>
<td>All the points are far away from the sphere due to distance from the center. It has no forces, bases, edges or vertices.</td>
</tr>
<tr>
<td>Prism</td>
<td>It has at least three faces each of which in a parallelogram. The upper and lower faces are called the bases of the prism. And its bases are perpendicular congruent polygons. The shape of the base represents the shape of the prism.</td>
</tr>
</tbody>
</table>
Determine the base for each shape then classify:

Circle, Cylinder          Triangle, Pyramid                           Triangle, triangular prism

Example (2)
Classify the composite solid shapes which is formed the composite solid and determine the number of faces and edges and vertices:
the shape consists of
* cone : one base , one vertex, no face , no edge.
* cylinder : two bases, no vertex, no edge.

Example (3)
Classify the composite solid shapes which is formed the composite solid and determine only the faces and vertices:
the shape consists of
* cube : six faces , eight vertices, 12 edges.
* sphere : no faces, no vertices.

Example (4)
Classify the composite solid shapes the tower in the picture is formed of, and determine the number of faces and edges of each shape.
The tower consists of
i) Rectangular prism (rectangular parallelepiped)
ii) Cube

Number of faces : 6
Number of edges : 12
Number of vertices : 8

NOTE : A rectangular parallelepiped and a cube both have the same number of faces, vertices and edges.

Example (5)
Determine the base for each shape then classify:

Circle, Cylinder          Triangle, Pyramid                           Triangle, triangular prism
To fill in the blanks

<table>
<thead>
<tr>
<th>solid shapes</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td>No. of faces</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of edges</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of vertices</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Determine base form of the followings and classify them.

Classify the composite solid shapes and determine the number of base faces, edges and vertices of the following:

Questions 1-7 are similar to examples 1-5

Solve the Exercises

Classify the composite solid shapes and determine the number of base faces, edges and vertices of the following:

Questions 1-7 are similar to examples 1-5
Solve the problems

**Education:** What is the shape which represents the Math book? and what is the number of vertices and faces and edges of the shape of maths book?

**School:** Search in the class for the geometrical figures and classify them.

**Space:** Determine the shapes in the pictures and classify them.

**Monuments:** Determine the number of vertices, faces and edges in each solid shapes in the pictures below.

Think

**Use the properties you have learnt about geometrical figures to compare between quadrilateral prism and quadrilateral pyramid.**

**What is the geometrical figure which has one base and only triangular faces.**

Use the geometrical figures to determine the following:

**Parallel planes**

**Intersecting lines**

**Two points on diagonal**

Write

Name of triangular figure which has curved face and has no vertex and edges.
**Coordinate Plane**

**Idea of the lesson:**
*Representation ordered pairs and recognize it in the coordinate plane.

**Vocabulary:**
* Coordinate plane
* X-axis
* Y-axis
* Origin point
* Coordinates of the point
* Ordered pairs

The coordinate plane consists of the intersection of two perpendicular straight lines. The horizontal straight line is known as the x-axis and one vertical on it is called the y-axis. They both intersect at a point called the origin.

**[ 5-3-1 ]** Distinguishing quadrants in a coordinate plane

The coordinate plane is divided into four parts known as quadrants; first quadrant, second quadrant, third quadrant, fourth quadrant.

**Example (1)**
Determine each points belong to which quadrant in the given coordinate plane.

i) Points A, B, C are located in the first quadrant
ii) Points D, E, F are located in the fourth quadrant

Link the points of the first and fourth quadrants, what do you notice? Two triangles have the same angles, sides and area.

**[ 5-3-2 ]** Representing point in the figure is triangle coordinate plane

Points are represented on a coordinate plane by ordered pairs. An ordered pair consists of two numbers (x, y). The first number x represents the x-axis (units to the right or to the left). The second number y represents the y-axis (units upwards or downwards).

**Example (2)**
Represent each ordered pair with a point on the coordinate plane and determine which quadrant each point is located at.

i) (5,3) Located in the first quadrant
ii) (-2,1) Located in the second quadrant
iii) (0,6) Located on y-axis
iv) (-5,-2) Located in the third quadrant
v) (3,0) Located on x-axis
vi) (4,-4) Located in the fourth quadrant
**Example (3)**

Determine the coordinates of each point in the given coordinate plane.

i) Coordinates of point A
Go from the origin to the left and count four units. A(-4, 0)

ii) Coordinates of point B
Go from the origin to the right and count two units and upward five units B(2,5)

iii) Coordinates of point C
Go from the origin to the left and count three units, then 5 units to downwards. C(-3,-5)

**Example (4)**

The map was divided to a coordinate plane where the x-axis represents the distance crossed to the right or to the left and y-axis represents the distance crossed upwards or downwards. Which city is the point (2, -2) located at? In which quadrant?

Start from the origin, move two units to the right and downwards to find the point (2, -2) at Al-Qadisiya City, it is in the fourth quadrant.

Start from the origin and move four units to the right and upward to find the point (0,4) in Kirkuk City, it is located on y-axis.

**Note:**
Determine at least two points at following (Basra, Ninawa, Anbar)
Determine the quadrant of each point belongs to, then write the ordered pairs.

1. A
2. B
3. C
4. D
5. E
6. F

Draw the coordinate plane on a graphing paper then represent each ordered pair with a point, name it and determine which quadrant or axis it is at?

7. (-3, -2)
8. (0, 1)
9. (-2, 0)
10. (4, -1)
11. (4, 3)
12. (5, -6)
13. (-2, 9)
14. (8, 0)

Questions 7-14 are similar to example 2

Solve the Exercises

15. Determine the quadrant of each point belongs to given coordinate plane and write the ordered pair of each point.

16. In the question (15) link the points in each quadrant what you note?

Draw the coordinate plane on a graphing paper then represent each ordered pair with a point, and name it, then determine the quadrant or axis it is at?

17. (-1, -2)
18. (0, -5)
19. (0, 4)
20. (6, 7)
21. (-2, 5)
22. (4, -6)
23. (6, 0)
24. (-1, -7)
25. (4, 5)
26. (7, -1)
27. (-6, -5)
28. (-7, 6)

Questions 1-6 are similar to example 1
Solve the problems

Represent the ordered pairs on the coordinate plane, and draw segments between the points (the dots) and name the resulting shape and which quadrant located in?

29 A (0,2), B(0,-2), C(2,2), D(2,-2)

30 E(-6,-3), F(-3,-1), G(4,-1), H(4,-6)

Geography: use the coordinate lines and determine some points which fixed the following:

31 Egypt
32 Sudan
33 Iraq
34 Algeria

Geometry: Graphically show four points on the coordinate axis that they form the vertices of a rectangle once they are linked together then determine the ordered pairs.

Think

36 Mention the x-axis of a point on y-axis.

37 Mention the y-axis of a point on x-axis.

38 Open problem: The quadrant where a point is located can be determined without the need to recourse with a graphical representation in a new way, give an example about it?

39 Challenge: Give the reason why the location of point (-2,2) is different from that of the point (2,-2).

Write

Three ordered pairs representing a right triangle when representing them on the coordinate axis.
The idea of the lesson
* Learning the translation, reflection and symmetry and drawing them.

**Vocabulary:**
* Translation
* Reflection
* Symmetry
* Axis of symmetry
* Line of reflection.

Symmetry is a property many things can be described with such as Symmetry shapes. A human is said to be symmetric as well since his right half is symmetric to his left half.

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### 5-4-1 Axis of Symmetry

A symmetric shape around one axis can be pleat on a straight line, as a result you get two congruent halves. The line of pleating in this condition is called the axis of symmetry. The symmetry around the y-axis for the point \((x, y)\) can be expressed as \((-x, y)\). And the symmetry around the x-axis for point \((x, y)\) can be expressed as \((x, -y)\).

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**Example (1)**

Is there an axis of symmetry for the human in the picture in Learn section?

A vertical axis of symmetry can be drawn to divide the body in the picture into two congruent halves where the right side of the human body is congruent to its left.

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**Example (2)**

Determine the axis of symmetry for the following shapes.

- **Horizontal Axis of Symmetry**
- **There are no axis of symmetry**
- **It has many axes of symmetry (Horizontal and Diagonal)**
- **Vertical Axis of Symmetry**
**Reflection**

Reflection: is the transfer of a shape to its opposite / reflected image for example, the shape of letter p according to a vertical line (reflection axis) or (a mirror) is q.

**Example (3)** In the given picture, compare the shape of the bird in the water.

i) Look at the points on the shape, what is their arrangement?
The points A, B, C are arranged in clockwise direction.
ii) How does their arrangement seem in the water?
They are now anti-clockwise.

**Example (4)** Copy the shape ABCD on a graphing paper then draw its image for a reflection across line shown.

i) Find the ordered pairs which represents each points of A B C D.
ii) Find the number of units between each vertex and the reflection line.
iii) Create a point to each vertex on the other side of the line with the same distance.
iv) Link the new vertices (points) to form a of its reflection image A B C D.

**Translation**

Translation: translating is a shape from one location to another without rotating it.
Translation can be expressed in the form of ordered pair (x+a,y+b) as for translating point (x,y) where a unit horizontally and b unit vertically.

**Example (5)** Copy the triangle ABC on a graphing paper then draw its translation image by 5 units to the left and 3 down.

i) Move each vertex of triangle ABC, 5 units right and 3 down. The translation of point A (2,1) is represented by (2-5, 1-3) in order to be A’ (-3,-2)
The translation of point B (6,1) is represented by (6-5, 1-3) in order to be B’ (1,-2)
The translation of point C (1,7) is represented by the ordered pair (1-5,7-3) in order to be C’ (-4,4)

ii) Link the new vertices to form the image of triangle after translation A’ B’ C’
Make sure of your understanding

Look at the shapes in the following pictures and mention their axes of symmetry? If it’s found!

1
2
3

4 Draw the shape A B C D whose coordinates are A(1,1), B(4,2), C(4,1), D(1,2)
Then draw its reflected image with respect to the x-axis and write the coordinates.

5 Point B was translated downwards with 2 units and 6 units to the right.
What are the coordinates of point B if point B was (2,1)?

Solve the Exercises

Use the shapes shown below, determine if the letter has an axis of symmetry, if it’s found.

6
7
8

Draw the triangle ABC whose coordinates are A(3,4), B(1,2), C(4,1) then draw its image
on a graphing paper and write its coordinates after making the following transitions:

9 Reflection with respect to the y-axis.

10 Translation of one unit to the left and 4 down.

11 Translation 3 units to the right and 2 units upward.

12 If the rhomboid was translated 4 units to the right and 1 upwards. What are the coordinates of point C?
Solve the problems

Copy these shapes on a graphing paper then draw reflection with respect to the line as shown. Describe the type of the symmetry if it’s found.

Describe the kind of translation for each shapes, if its found.

Hand works: Show reflectinos for some shapes in the following pictur and show the translation for some shapes in the same pictur.

Geometry: A circle whose radius is 5 units and its center is the point (3,3). If a translation was done by 2 units upwards and 3 units to the left. What are the new coordinates of the center?

Think

Numerical Sense: Draw the octagonal on the graph paper then draw its reflection with respect to the y-axis. Write its coordinates after the reflection. The coordinates are A (2,2), B (3,2), C (N 1,3), D (1,4), E (2,5), F (3,5), G (4,4), H (4,3)

Challenge: Suppose that point A (-3,5) is an image of point A (3,5) in the reflection, which axis was the reflection made up on? Find the reflection without graphing according to which axis. Explain the answer

Write

A real-life problem where translation is used on a certain shape then solve the problem.
Congruence and Similarity

Idea of the lesson
Determining the congruent shapes, and distinguishing the similar shapes, and finding the measure of an angle, and the unknown length in a pair of congruent shapes.

Vocabulary:
Congruence
Similarity
Corresponding angles
Corresponding sides

Learn

Look at the shapes, which of the squares in the middle is bigger than the other?

Are all the blue squares congruent to the red ones?

[ 5-5-1 ] Congruence

Congruence; is the equality of sides and angles of a polygon with those of another polygon. The symbol of congruence is ‘≅’.

Example (1)
In LEARN section above, both squares are congruent the apparent difference is only a visual trick, to check that you can simply put the first square on the second to notice the complete congruence. While the rest of the squares in the two sketches are similar to one another since their corresponding lines are proportional.

Example (2)
Distinguish the congruent shapes in each picture

In the picture above all shapes are congruent.

All toys in the picture are not congruent.

The group of balls are not congruent, since each one has different shape and size.
[5-5-2] Congruence Polygons

- Two triangles are congruent if their sides are congruent.
- Shapes which contain more than three sides must be congruent in sides and measure of their angles.
- If two polygons are congruent we can find the measure of an unknown angle or length of an unknown side.

Example (3)
Are the two triangles congruent?
If each side of the first triangle matches each side of the second then both triangles are congruent.

Example (4)
Find the measurement of the unknown angle for the given shape.
If each side of the first triangle matches each side of the second then both triangles are congruent.
Since both shapes are congruent then the measurement of their angles are congruence as well. So the measurement of the unknown angle in the figure is 66°.

[5-5-3] Similarity
Shapes having the same shape and their corresponding sides are proportional. The symbol of similarities is ‘~’.
- Their sides are proportional.
- The measure of their angles are congruence.

Example (5)
Which of the following quadrangles are similar to rectangle ABCD?
In order the quadrangles to be similar their corresponding sides must be proportional such that:
The shape of the blue quadrangles is not similar to the shape of the yellow quadrangles since the length of their sides are not proportional.
The shape of the blue quadrangles is similar to the shape of the red quadrangles since their sides are proportional.
Make sure of your understanding

Distinguish the following congruence shapes:

1

2

3

Find the measurements of the unknown angles and lengths of the unknown sides in the following congruency shapes:

4

5

Questions 1-3 are similar to example 2

Questions 4-5 are similar to example 4

Solve the Exercises

Distinguish the congruence shapes:

6

7

8

Find the measurements of angles, and lengths of the unknown sides the following congruency shapes.

9

10

24
Solve the problems

Distinguish the following congruence shapes:

11 12 13

Determine whether the following polygons are congruent or not.

14 15 16

17 Geometry: In the following figure, a building with a height of 18 m forms a shadow of 12 m length, what is the height of nearby tree forming a shadow of 3 m length?

Think

18 Challenge: If the triangle \( \triangle ABC \sim \triangle DEF \), what’s the length of \( AB \)?

19 If the triangle \( \triangle ABC \sim \triangle CDE \), what’s the length of \( AB \)?

Write

When are the similar polygons congruent?
**The idea of the lesson**
Solve the problems by using a plan (make a model)

**Learn**
Zeina tried arranging 3 cubes in different ways. What are the possible ways to arrange the four cubes in different ways and side by side?

**UNDERSTAND**
What are the givens in the problem? 3 colour cubes that must be arranged side by side.
What is asked in the problem? Number of ways possible in order to do that.

**PLAN**
How to solve the problem? Make a model to clarify those different ways of arranging the cubes.

**SOLVE**

1. ![Arrangement 1](image1)
2. ![Arrangement 2](image2)
3. ![Arrangement 3](image3)
4. ![Arrangement 4](image4)
5. ![Arrangement 5](image5)
6. ![Arrangement 6](image6)

**CHECK**
Check the above pictures which among all possible different ways of arranging the cubes.
1. **Patterns:** How many squares are there in the shape No.8 according to the following pattern?

   ![Shapes](image)

   shape (1)  shape (2)  shape (3)  shape (4)

2. **Basketball:** 6 friends came together to form a basketball team, how many teams can be formed of 5 players?

3. **Colours:** Sana needs two colouring kits to draw 5 paintings. What is the least number of paintings she can draw/paint with 10 kits?

4. **Kitchen tools:** Aleen has 6 coloured cups, she wanted to offer juice in 5 different cups, how many times she can do it?
### Vocabulary

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### 5-1 Regular polygons and Interior, Exterior and Central Angles.

**Example (1):** Find the measure of the exterior angle in a regular trine

Write the rule: \[
\frac{360}{n}
\]

Substitute “n” for the number of sides which is 3.

So, the measure of exterior angle in trine is 120°.

**Example (2):** Which polygon has 540° as the sum of its interior angles?

\[
180° (n - 2) = 450°
\]

\[
n - 2 = 3
\]

\[
n - 2 + 3 = 5
\]

So, the polygon is a pentagon.

**Exercise (1):** Find the measure of the exterior angle of a polygon (12 sides)

**Exercise (2):** Find the polygon whose interior angles’ measurement is 90°.

**Exercise (3):** Which polygons are concave and which are convex?

![Shapes]
[5 - 2] Polyhedron Forms and Composite Polyhedron Forms

Example: Draw each of the following and mention the number of faces in each shape.

i) Hexa Pyramid
   Number of faces: 7

ii) Septuple Prism
   Number of faces: 9

Exercise (1): Find the number of the faces and edges of the following:
   i) Tripartite Pyramid
   ii) Octa-prism

Exercise (2): Determine the shape of the base cube of the following:
   i) Cube
   ii) Cone

Exercise (3): Determine the shape of the base and the names of the followings.

[5 - 3] Coordinate Plane

Example: Determine the quadrant each points.
Points A,B,C,D are located in the first quadrant.
Points F,E,H,O are located in the second quadrant.

Exercise (1): Represent each point on the coordinate plane and link the points mention the resulting shape and quadrant.
A(4,5), B(2,5), C(3,7), D(2,0), E(4,0)

Exercise (2): Determine the coordinates for each point on the coordinate plane below.
**[5 - 4] Translation, Reflection and Symmetry**

**Example(1):** Draw the following shapes in a grid and translate it 8 units in the direction of negative x-axis.

**Exercise(1):** Draw the following shape on a grid and translate it six units in negative x-axis and four units in negative y-axis.

**Example(2):** Find if the following shapes are reflection axes and draw all axes of reflection.

- has vertical reflection axes
- has no reflection axes
- has several reflection axes

**Exercise(2):** Find if the following shapes are reflection axes and draw all axes of reflection.

⇒ م ج

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**[5 - 5] Congruence and Symmetry**

**Example(1):** Distinguish the congruent figures in the shape.

**Exercise(1):** Distinguish the congruent four-sided figures in the shape.

**Example(2):** Find the measure of the missing angle in the shape.

**Exercise(2):** Find the measure of the missing angle in the shape.
1. What is the polygon which the sum of its interior angles 1980° and what is the magnitude of its exterior angle?

2. What is the polygon which the sum of its interior angles 1800° and what is the magnitude of its exterior angle?

Which one of the polygon is concave or convex.

3. 

4. 

5. 

6. Determine the ordered pair for each point and the quarter.

7. Find the number of faces and edges for:
   - hexagon pyramid
   - heptagon prism

8. Copy the shape on a graphing paper and then draw its reflection image and translation image by 5 units upwards.

9. Draw the symmetry axes and name it if its found.

10. 

11. 

12. 

13. Find the length of missing side.

14. Draw a hexagon on a grid paper and determine its points and divide it into congruent triangles and find the number of triangles.
Ziggurat UR oldest temple in the city of Nasiriyah rised above the ground by three layers the base dimensions of the first layer $34m \times 26.5m$ with height $11m$ above the level of the curtyardal and the secound layer height is $6m$ and the third layer was disappeared and what had left of landmarks is $3m$ only.

the first volume of the layer $= 11m \times 34m \times 26.5m$

$= 9911m^3$
Calculate the area of the parallelogram whose base length is 20cm and height is 5cm.

Calculate the area of a trapezoidal, the length of both of its basis are 6cm and 8cm, its height is 5m.

Calculate the circumference of a circle whose radius is 14cm.

Calculate the area of a circle whose radius is 10cm.

The area of a circle is 616cm², calculate its circumference.

The circumference of a circle is 132cm, calculate its area.

Find the area of each of the following.

A circular garden with radius 25m is surrounded by a paved gangway of 5m width. Calculate the area of the gangway and its (outer) circumference.
Rate effect (scale) on the perimeter and area (enlargement and reduction)

**Idea of the lesson**
The enlargement and reduction effect of geometrical shapes on their perimeters and areas.

**Vocabulary**
- Dilation
- Center of dilation
- Dilation coefficient
- Enlargement
- Reduction

**Learn**
The following figure shows a grid. Each square on the grid is 2cm. Re-draw the irregular pentagon using a grid whose each square side is 4cm in length. Use point “A” as a start point.

Measure the length of the sides in the figure in both cases. Compare them and conclude the relation between two measurements.

Now, guess the length of the side of the square paper in the grid to make a second copy of the shape such that its dimensions would be half of those of the first copy (original shape).

**[6-1-1] The Concept of Extensibility and Center Dilation**

**Dilation**: is the resulting from enlargement or reduction a given shape. The image is similar to the original one and has symmetrical dimensions. The center of dilations is one of the points in the original shape. When measuring we fix it in order to adjust the shape measurements.

**Dilation**: coefficient is the ratio of the image’s dimensions to the original shape dimensions.

**Example (1)**

Draw the dilation, draw a circle with center “C” and let its radius be 3 squares of the paper using the compass.

Drawing a picture of this circle is asked on another grid of the same squares size using an dilation of center “C” and \(\frac{1}{3}\) as coefficient.

**Step 1**: We choose a point in the middle (at the center) of the new squares paper and we show it as and name it “C” as a center of dilation.

**Step 2**: Focus the needle of the compass on one of the points of the squares in the grid and we move the second arm by one square.

**Step 3**: We focus the compass on point “C” and move the other arm to draw the new circle.

**Step 4**: Using the compass, calculate the length of the new circle’s radius. You will find that it is one square. As a conclusion, the circle’s radius was dilated by coefficient \(\frac{1}{3}\).
**Example (2)**

Let ABCD be a rectangle whose vertices are A(1,1), B(4,1), C(4,3) and D(1,3). Draw it then draw an dilation to it, let its centre be origin point and its coefficient be 2.

Obviously the dimensions of the rectangle are 2,3 of the squared units. Draw rectangle A’B’C’D’ with dimensions 4,6.

Notice that:
\[
\frac{A'B'}{AB} = \frac{6}{3} = 2, \quad \frac{C'D'}{CD} = \frac{6}{3} = 2
\]
\[
\frac{A'D'}{AD} = \frac{4}{2} = 2, \quad \frac{B'C'}{BC} = \frac{4}{2} = 2
\]

Such that:
- A (1, 1) → A’ (2, 2)
- B (4, 1) → B’ (8, 2)
- C (4, 3) → C’ (8, 6)
- D (1, 3) → D’ (2, 6)

So, rectangle A’B’C’D’ is enlargement in of the rectangle ABCD and the coefficient of the dilation is 2.

**Example (3)**

**TV screen:** The TV screen is characterized by its ability to change the image dimensions such that if the original image dimensions shown on the screen are 100cm, 52cm. If we reduction the show by an dilation coefficient of \(\frac{3}{4}\), how would the perimeter and image area be?

\[
P = 2 \times (L+W) = 2 \times (100+52) = 304 \text{ cm}
\]
\[
A = L \cdot W = 52 \times 100 = 5200 \text{ cm}^2
\]

And when reduction is applied the image dimensions become as follows

\[
L = \frac{3}{4} \times 100 = 75 \text{ cm}, \quad W = 52 \times \frac{3}{4} = 39 \text{ cm}
\]

Such that:
- \(P' = 2 \times (39 + 75) = 228 \text{ cm}\)
- \(A' = L \cdot W = 39 \times 75 = 2925 \text{ cm}^2\)

Notice that:
\[
\frac{P}{P'} = \frac{304}{228} = \frac{3}{4}, \quad \frac{A}{A'} = \frac{5200}{2925} = \frac{9}{16} = \frac{3}{4}
\]
If we check examples 2 and 3 we conclude that:

- The dilation whose coefficient is greater than 1 leads to enlargement that the shape’s dimensions are dilated by the same amount of the dilation coefficient.
- The dilation whose coefficient value is between 0 and 1 leads to reduction that shape’s dimensions are dilated out by the value dilation coefficient.
- The dilation coefficient of the perimeter (enlargement or reduction) is the same as the expansion coefficient of the shape’s dimensions.
- The dilation coefficient of the area (enlargement or reduction) is the square of the dilation coefficient of the shapes’ dimensions.

Make sure of your understanding

1. On a grid, graphically show a right triangle whose vertices are A(2,1), B(2,4), C(5,1), find its area, then graphically show the triangle resulting from enlargement in by the coefficient 3.

2. On a grid, graphically show a circle whose centre is the origin and its radius is 3cm. Calculate its perimeter and area. Draw an image of it under the effect of:
   A) enlargement in with coefficient 2
   B) reduction with coefficient \( \frac{1}{3} \) and calculate its perimeter and its area.

Solve the Exercises

3. On a grid, draw the following shapes then draw their dilation by using the given information.

   dilation center A
dilation center B
dilation center C

   coefficient of dilation 2 coefficient of dilation \( \frac{1}{4} \) coefficient of dilation \( \frac{2}{5} \)

4. Find the coordinates resulting from the expansion of the quadrilateral shape ABCD in each of the following cases using the given expansion coefficient if the center of dilation is the origin point.
   i) A (0, 3), B (2, 1), C (0, -2), D (-2, -1) coefficient of dilation = 4
   ii) A (-3, 2), B (3, 3), C (5, -2), D (-2, 0) coefficient of dilation = \( \frac{2}{3} \)

5. If you knew one of the polygons in the given figure is an exdilation of the other. Find the coefficient and show whether it is enlargement or reduction.
Solve the problems

6. Abdulla designed a scheme of a part of the Freedom Statue with dimensions of 50cm, 15cm. If he wanted to reduction the scheme using the coefficient \( \frac{1}{5} \), calculated the perimeter and the area of the reduction scheme.

7. Design: Ahmed is a construction engineer who is used to fulfilling the projects of building residential buildings in his daily notebook, and when the design is finished he dilates it in by a coefficient of 10 on a maps paper are 3.5 cm, 4.8 cm. If the dimensions of the bedroom on the map. What is its area and what is the perimeter?

8. The convex lens is used as a magnifying lens, if a body is put between the lens and its focus. If you take a look at the letter “M” written on the paper whose height is 5mm. The lens’ reduction is \( \frac{3}{5} \). What is its height after being dilated?

Think


10. Open Problem: Graphically represent a square, then show an image of it under the effect of dilation with coefficient greater than 1. Then represent the resulting image dilated by a coefficient less than 1. Expect the value of dilation coefficient between the original shape and the resulting shape. Explain why and check your answer.

11. Numerical sense: What do you expect the image of the shape will be if it was under the effect of dilation as in the following cases;
   i) the coefficient is 1
   ii) the coefficient is 0

Write

A general mathematical formula to find the new coordinates of the ordered pair \((x,y)\) under the effect of dilation by coefficient “k”.
Idea of the lesson
Getting to know how to extract the volume of a cube and a parallelepiped.

Vocabulary
* Solid
* Prism
* Edge
* Vertex
* Face
* Length
* Width
* Height.

An solid shape like the table and the cube is characterized by having three dimensions unlike the standard shape (plane shape) which has only two dimensions. The three dimension are: length, width, height.

In the language of Geometry science, each body of many surfaces has two parallel regular congruent polygons called (the regular prism).

A prism is classified according to its base shape. So the prism is quadrilateral if its base had a quaternary shape as in the cube and cuboid (parallelepiped).

[ 6-2-1 ] Cube

The cube is a body whose 6 faces are squares. It can also be called a quadrilateral prism, its height is equal to its base side. In other words the three dimensions of the cube are equal.

As in details: The cube consists of six equal faces, eight vertices or angles, and twelve edges.

Volume of a Cube

Volume = edge length x edge length x edge length

\[ V = L \times L \times L \]

Where V is volume of the cube and L represents the cube edge.
Example (1) Find the volume of a cube whose edge length is 6cm.

Use the cube volume rule in symbols

\[ V = L \times L \times L \]

substitute and simplify

\[ V = 6 \times 6 \times 6 = 216 \text{ cm}^3 \]

so the volume of cube is 216 cm\(^3\)

Example (2) The rubik’s cube consist of 27 different colored tiny cubes, the tiny cube’s side is 1.9 cm and we can calculate the volume of the rubik’s cube in two ways.

First way:

1.9 \( \times \) 3 = 5.7 cm

the volume of the rubik’s cube is:

5.7 \( \times \) 5.7 \( \times \) 5.7 \( = \) 185.193 cm\(^3\)

Second way:

the volume of the tiny cube is:

1.9 \( \times \) 1.9 \( \times \) 1.9 = 6.895 cm\(^3\)

so the volume of the rubiks cube is

27 \( \times \) 6.859 = 185.193 cm\(^3\)

Example (3) A cubed water tank whose edge is 3m, water is poured into it with a rate of 9m per hour. Calculate the time needed for it to be filled.

The capacity of the tank means how much water it can hold in and it is exactly the volume of the cubed tank.

\[ V = L \times L \times L \]

\[ V = 3 \times 3 \times 3 \]

\[ V = 72 \]

So tank capacity equals; 27 m\(^3\)

The water is poured inside the tank with a rate of 39 m/h. So the time needed for the tank to be filled can be concluded by dividing the tank capacity by the water flow rate;

\[ 27 \div 9 = 3 \text{ hours} \]
A parallelepiped is a body whose both of its bases are rectangular and all of its side faces are rectangular as well. and each two opposite faces in it are congruent. Its volume is measured as follows;

\[ V = \text{base area} \times \text{height} \]

\[ V = L \times w \times h \]

Where “L” is base length, “w” is base width, and “h” is height

**Example (4)** Find the volume of the parallelepiped whose dimensions are 3cm, 4cm, 8cm.

Use parallelepiped volume rule in symbols;

\[ V = L \times w \times h \]

Substitute and simplify

\[ V = 8 \times 4 \times 3 = 96 \]

So parallelepiped volume is 96 cm³

**Example (5)** A parallelepiped of squared base whose edge length is 8cm and its height is 10cm. Find its volume.

Use parallelepiped volume rule in symbols;

\[ V = L \times w \times h \]

Substitute and simplify

\[ V = 8 \times 8 \times 10 \]

\[ V = 640 \]

So parallelepiped volume is 640 cm³

**Example (6)** Fishbowl: The bowl is in the form of a parallelepiped whose dimensions are; 1m, 0.5m, 1.5m.

Find the volume of water needed to completely fill it.

\[ V = L \times w \times h \]

\[ V = 1.5 \times 0.5 \times 1 \]

\[ V = 0.75 \]

So the volume of water needed to fill the bowl is 0.75 m³
What is the edge length of a cube whose volume is 1000cm\(^3\)?

What is the height of a parallelepiped whose volume is 48cm\(^3\) and base area is 12cm\(^2\)?

A parallelepiped of a squared base and 10cm height. If its volume is 90cm\(^3\), what’s the side length of its squared base?

A cubed water tank whose edge length is 3m contains an amount of water reaching the height 1.5m. Calculate the amount of water which needs to be added for it to completely fill it.

Calculate the volume of the lower shadowed part of the bird house in the figure.

Challenge: A sweets company decided to change the shape of the box from a parallelepiped with the dimensions 4cm, 2cm, 8cm to a cubed box in a way that the box’s capacity remains constant. Find the length of the edge of the cubed box.

Open Problem: Three cubes of edge length 5cm were put side by side. What is the resulting shape and its volume?

Numerical Sense: Which of the following has a better effect on the parallelepiped? A) Doubling its base area. B) Doubling its height. Explain your answer with an example.

A general mathematical formula for the volume of the parallelepiped whose length is \((k)\)cm and width is half of the length and its height is three times of its length.
Lateral area & total area of three dimensional geometric shapes (Cube/Parallelepiped)

Idea of the lesson
Know how to find the side surface area and total surface area of the cube and the parallelepiped.

Vocabulary
* Cube edge length
* Length
* Width
* Height.

Learn
The lateral area of a cube is the area of 4 squares. While the total parallelepiped area is the area of 6 squares.
The lateral area of a parallelepiped is the area of four rectangles. Each two opposite rectangles have the same area. And the total total area of a parallelepiped is the area of six rectangle. The total area is sum of the upper base and lower base.

[ 6-3-1 ] Cube

Lateral: is the area of the four faces which are located on the sides of the geometric shape or it is the area of four squares, so;

Lateral area = 4 x edge length x edge length
LA = 4 \times L \times L

Where A is the lateral area and “L” is the cube edge length.

Total area: is the area of the six squared faces located on the surface of the geometric shape. Or it is the area of six squares, so;

Total area = 6 x edge length x edge length
TA = 6 \times L \times L

Where TA is the total area and “L” represents the cube edge length.

Example (1)
Find the lateral area and the total area of a cube whose edge is 12cm.
Use cube’s lateral area rule by symbols
LA = 4 \times L \times L
Substitute and simplify
LA = 4 \times 12 \times 12 = 576

So the lateral area of the cube is 576cm²
Use cube’s total area rule by symbols
TA = 6 \times L \times L
Substitute like tuke and simplify
TA = 6 \times 12 \times 12 = 864
So the total area of the cube is 864cm²
Example (2)

A cube whose surface is $16\text{cm}^2$. Find;
1- Its edge length  
2- Its volume

Write the rule of cube lateral area
$LA = 4 \times L \times L$
Substitute
$16 = 4 \times L \times L$
Use multiplication and division theory $L \times L = \frac{16}{4}$
Then $L($edge length$) = 2\text{cm}$

Write the cube’s volume rule $V = L \times L \times L$
$V = 2 \times 2 \times 2$ Substitute
$V = 8\text{cm}^3$ Simplify

So the cube’s volume is $8\text{cm}^3$

Parallelepiped

Lateral Area: is the sum of areas of two couples of side rectangles.

$Lateral \ Area = base \ perimeter \times height$

$= 2 \times (length + width) \times height$

$LA = 2 \times (L + W) \times h$

Where $A$ represents lateral area, $L$ is base length, $W$ is width and $h$ is height.

Total Area: is the sum of areas of six rectangular faces that represent the geometric shape surface. Or it is the sum of lateral area plus the sum of two bases’ area. So;

$Total \ Area = lateral \ area + two \ bases \ area$

$TA = [2 \ (L+w) \times h] + [2 \times L \times w]$

Where $TA$ represents the total area, $L$ is the base length, $W$ is the base width, $h$ is height.
Example (3) Find the total area of a parallelepiped whose dimensions are 3cm, 4cm, 8cm.

Lateral area rule of a parallelepiped in symbols
\[ L_A = 2(L+w) \times h \]
Substitute and simplify
\[ L_A = 2(8+4) \times 3 = 2 \times 12 \times 3 = 72 \]
So, the side area of the parallelepiped is 72cm²

Use the total area rule of the parallelepiped in symbols;
\[ T_A = 2(L+w) \times h + 2 \times L \times w \]
Substitute and simplify
\[ T_A = 2(8+4) \times 3 + 2 \times 8 \times 4 \]
\[ = 2 \times 12 \times 3 + 64 = 136 \]
So, the total area of the parallelepiped is 136 cm²

Example (4) Matches box: the box is in the form of a parallelepiped whose dimensions are 5cm, 3cm, 2cm. Calculate the area of the carton board needed to make it.

\[ T_A = 2(L+w) \times h + 2 \times L \times w \]
\[ T_A = 2(5+3) \times 2 + 2 \times 5 \times 3 \]
\[ T_A = 32 + 30 = 62 \]
So, the area of the carton board needed to make the matches box is 62 cm².

Make sure of your understanding

1. A parallelepiped base length is 4cm, its width is 8cm and height is 8cm. What is its volume?
2. A cube’s edge length is 2.4 cm. What is its volume?
3. A parallelepiped’s height is three times its base length. If its base length is 4cm and width is 8cm, what is its volume?

Questions 1-3 are similar to examples 3,4

Solve the Exercises

4. A parallelepiped whose dimensions are 3cm, 5cm and 6cm. Calculate each of its lateral area and total area.
5. A parallelepiped whose base is square has edge length of 5cm. Its height is 10cm. Calculate the lateral area and total area of the parallelepiped.
6. A parallelepiped whose lateral area is 49cm² and its base perimeter is 12cm. Find its height.
7. A parallelepiped whose base is square has height of 10cm. If its total area is 280cm², find the edge length of its square base.
Swimming pool: whose base dimensions are 4m, 8m and height is 2.5m is wanted to be paved with square pieces of ceramic with edge length 0.2m. Calculate the number of pieces needed.

Furniture: A wooden locker with a frontage consisting of six drawers, rectangular and equal in area. Each drawer’s dimensions are 0.5 m and 1m. If the locker’s width is 2m, find its lateral and total area.

Challenge: 45 parallelepiped wooden pieces were placed together side by side. Each parallelepiped has a square base with edge length 2cm, and height 3cm as shown in the figure. Calculate the total area of the geometric shape.

Open problem: Consider a parallelepiped water tank with a square base and height five times of its base. If you know that the cubic volume of water inside it as 40m³, what is the height of the tank?

Numerical sense: which of the following has a greater effect on the lateral area of a parallelepiped; 
A- doubling its base area 
B- doubling the height

Explain your answer by a numerical example.

A general mathematical formula of the total area of a parallelepiped whose length is (k) cm, its width is half of its length, and its height is three times of its length.
Rate effect (scale) on the volume and surface area
(enlargement and reduction)

**Idea of the lesson**
The enlargement and reduction effect of geometric shapes on their volume and surface area.

**Vocabulary**
- Dilation
- Dilation coefficient
- Enlargement
- Reduction

**Learn**
You have a cube whose edge length is 1 cm and another cube whose edge length is 3 cm.
> Find the volume and the surface area for both.
> Compare the volumes of both cubes then conclude the relation between them.
> Compare both areas then conclude the relation between them.

**Review of concept of Extensibility duality and center dilation**

Dilation: is the image resulting from reduction or enlargement in out a given shape. The image is similar to the original one and has symmetrical dimensions.
The center of dilation: is one of the points in the original shape. When measuring, we fix it in order to adjust the shape measurements.

**Dilation**: is the ratio of the image dimensions to the original shape dimensions.

**RULE 1:**
If V represents the original cube volume and V’ is the new volume after magnifying the edge length by dilation coefficient k, so;

\[
V' = k^3 \times V = (k \times k \times k) \times V
\]

**RULE 2:**
If A represents the area (whether lateral or total) of the original cube (TA)’ represents the new area after magnifying the edge length by the dilation coefficient k, so;

\[
A' = k^2 \times A = (k \times k) \times A
\]

**Example (1)**
We try to check these two rules using the givens in “Learn” section.

Original cube volume => \( V = (L \times L \times L) = (1 \times 1 \times 1) = 1cm^3 \)
Total area of the original cube => \( TA = 6 \times L \times L = 6 \times 1 \times 1 = 6cm^2 \)
When magnifying the edge length by coefficient of dilation (3), the edge length becomes 3×1 = 3 cm
Cube volume after dilation => \( V' = (L \times L \times L) = (3 \times 3 \times 3) = 27cm^3 \)
\( (TA)' = 6 \times L \times L = 6 \times 3 \times 3 = 54 \text{ cm}^2 \) Total area of the cube after dilation
**RULE 3:**
If $V$ represents the original volume of the parallelepiped and $V'$ is the new volume of it after being magnified by dilation coefficient $k$, so;

$$V' = k^3 \cdot V = (k.k.k) \cdot V$$

**RULE 4:**
If $A$ represents the area (lateral or total) of the original parallelepiped and $(A')$ represents the new area after magnifying by the dilation coefficient $k$, so;

$$A' = k^2 \cdot A = (k.k) \cdot A$$

---

**Example (2):** A parallelepiped of dimensions 15cm, 12cm, 30cm. A similar model was made with dilation coefficient $k = \frac{1}{3}$. Using the rules 3 and 4, find the volume and the lateral area of the mini model.

$$V = L \times w \times h = 15 \times 12 \times 30 = 5400\text{cm}^3$$

$$L \cdot A = 2 \times (L + W) \times h = 2 \times (12 + 30) \times 15 = 1260\text{cm}^2$$

$$V' = k^3 \cdot V = \left(\frac{1}{3}\right)^3 \cdot 5400 = \frac{5400}{27} = 200\text{cm}^3$$

$$(L \cdot A)' = k^2 \cdot (L \cdot A) = \left(\frac{1}{3}\right)^2 \cdot 1260 = \frac{1260}{9} = 140\text{cm}^2$$

---

**Example (3):** Money box: a minimized model of a money box was made in the shape of a cube whose edge length is 8cm. If greater model was wanted to be made by dilation coefficient $\frac{3}{2}$, calculate the volume and lateral area of the magnified model.

Model volume before magnifying
$$V = (L \times L \times L) = (8 \times 8 \times 8) = 512\text{cm}^3$$

Lateral area before magnifying
$$L \cdot A = 4 \times L \times L = 4 \times 8 \times 8 = 256\text{cm}^2$$

Volume & lateral area after magnifying
$$V' = k^3 \cdot V = \left(\frac{3}{2}\right)^3 \cdot 512 = 1728\text{cm}^3$$

$$(L \cdot A)' = k^2 \cdot (L \cdot A) = \left(\frac{3}{2}\right)^2 \cdot 256 = \frac{1260}{9} = 576\text{cm}^2$$
A cube whose edge length is 5cm was dilated (enlarged) by coefficient 3. Find its volume and total area after dilation.

A parallelepiped whose dimensions are 6cm, 3cm, 9cm. Find its volume and its lateral area under the effect of dilation coefficient of $\frac{2}{3}$.

If you know the lateral area of a cube is 32cm$^2$ and that it is under the effect of reduction that its lateral area was reduced to become 8 cm$^2$. Find the coefficient.

A parallelepiped with a square base of 3 cm edge length and a height five times greater than the edge length. Find its volume, lateral area, total area. Then find the following;

i) Its volume under the effect of dilation by coefficient 4.

ii) Its lateral area under the effect of dilation by $\frac{1}{6}$.

iii) Its total area under the effect of dilation coefficient of $\frac{1}{13}$.

A piece of iron is in the shape of a parallelepiped with a square base which has edge length of 20cm and height of 120 cm, if reduced it by the coefficient $\frac{1}{10}$, find its volume after the reduction.

Questions 1-4 are similar to examples 3,4

A parallelepiped with a square base of 3 cm edge length and a height five times greater than the edge length. Find its volume, lateral area, total area. Then find the following;

i) Its volume under the effect of dilation by coefficient 4.

ii) Its lateral area under the effect of dilation by $\frac{1}{6}$.

iii) Its total area under the effect of dilation coefficient of $\frac{1}{13}$.

A cube whose edge length is 2cm. Find its volume, lateral area and total area. Then find the following;

i) Its volume under the effect of dilation by coefficient $\frac{1}{4}$.

ii) Its lateral area under the effect of dilation by 5.

iii) Its total area under the effect of dilation coefficient of $\frac{5}{9}$.

If you know the total area of parallelepied is 13cm$^2$ and its total area under effect of dilation (enlarging) is 52cm$^2$. Find the coefficient of dilation.
Solve the problems

8. A cube of ice whose edge length is 5cm melts keeping its geometrical shape under the effect of dilation coefficient \( \frac{1}{5} \) in each second. Calculate the time takes to completely melt it.

9. The cake dough was put in a parallelepiped shaped metal mold. Its base dimensions are 30cm and 25cm. The cake dough’s height was 2cm. After getting it out of the oven, its height increased by dilation coefficient 4. Calculate its volume.

10. **Sponge**: A piece of sponge whose base dimensions are 100mm, 60mm, and the height is 20mm dipping into water and its dimensions are dilated by coefficient of 1.5. Calculate its total area after dilation.

Think

11. **Challenge**: How can you turn a cube whose edge length is 3cm to a parallelepiped of the same volume by applying two different dilations on two of its dimensions?

12. **Open problem**: What would the coefficient of a parallelepiped dilation be, if the exdilation effect on its dimensions was as follows:
   - length with the dilation coefficient 3
   - width with the dilation coefficient 5
   - height with the dilation coefficient 7

13. **Numerical sense**: If we enlarge a cube under the effect of dilation coefficient of 5 then we reduced the resulting body by the dilation coefficient of \( \frac{1}{3} \). What do you expect the final coefficient to be?

Write

A general mathematical formula to find the total area of a parallelepiped. The edge length of its base is \( x \)cm. Its height is three time greater than its base edge length under the effect of dilation coefficient of \( \frac{1}{3} \).
Surface area and Volume of compound three dimensional shapes

**Idea of the lesson**
Finding the surface area and volume of compound geometrical shapes.

**Vocabulary**
* Simple three dimensional shape
* Compound three dimensional shape

Two similar cubes whose each edge is 5cm were put one over the another as shown in the figure so a compound three dimensional shape was formed. It is determined. The two simple three dimensional shapes which composed it. Find the surface area and the volume of the compound shape by multiplying the surface area of the cube and its volume x2.

Find the volume and the surface area of a compound plane shape

Important note: When finding the surface area of a compound body. Attention must be paid to the surfaces is common between the adjacent three-dimensionals where they are counted only one.

- The compound shape is composed of two simple shapes or more. This is why we divided it to simple pieces.
- We calculate the volume and the surface area of each simple shape alone.
- We add volumes of the simple shapes get the volume of the compound.
- We add the surface areas of the simple shapes minus the common areas to get the surface area of the compound.

**Example (1)**
We try to find the volume and the surface area of the compound shape in “LERAN” section.

The two simple solids that form the compounds are similar cubes. The edge length of each cube is 5cm. We find the surface area and the volume of the cube as follows;

\[
V = L \times L \times L = 5 \times 5 \times 5 = 125 \text{ cm}^3
\]

\[
T A = 6 \times L \times L = 6 \times 5 \times 5 = 150 \text{ cm}^2
\]

We calculate the volume of the compound shape;

\[
V = 125 \times 2 = 250 \text{ cm}^3
\]

To calculate the surface area of the compound three dimensional shape the area of the common face must be subtracted which is the base of one of the cubes ans equals;

\[
S A = 2 \times 150 - 2 \times 25 = 300 - 50 = 250 \text{ cm}^2
\]

So the surface area of the compound shape is 250 cm²
In the following figure, notice that the compound shape can be divided into three. Three simple shape, one cube and two parallelepipeds in equal dimensions.

**First:** The compound shape volume. We find the volume of the cube the parallelepiped.

\[ V = L \times L \times L = 2 \times 2 \times 2 = 8 \text{ cm}^3 \]

\[ V = L \times w \times h = 5 \times 2 \times 6 = 60 \text{ cm}^3 \]

The total volume is:

\[ V = 8 + 2 \times 60 = 8 + 120 = 128 \text{ cm}^3 \]

**Example (3)**

Buildings: The building in the picture consists of 4 symmetric floors and it can be considered a compound shape which consists of 4 cubes placed one over on the another. If we knew the cube’s edge length (which represents each floor) is equal to 2.5m.

What is the total volume of the buildings?

\[ V = 4 \times (L \times L \times L) \]

\[ V = 4 \times (2.5 \times 2.5 \times 2.5) = 62.5 \text{ m}^3 \]

The lateral area of the building equals (the lateral area of each floor x 4)

\[ LA = 4 \times (4 \times L \times L) = 4 \times (4 \times 2.5 \times 2.5) = 100 \text{ m}^2 \]
If you knew that the compound shape consists of 8 match boxes with the dimensions 2cm, 4cm, 6cm. What is the total volume of the shape?

The dimensions of the upper board of the table are 1.2 m, 0.8 m, 1.2 m. The dimensions of each drawer of the six drawers are 1.2 m, 0.8 m, 0.3 m. Calculate the total volume of the table.
A table’s upper board has the dimensions 1m, 2m, 30cm. Its legs are parallel. The surface of its base is squared, its edge length is 25 cm and its height is 1m.

The following design is of a wooden table composed of three similar wooden pieces in the shape of a parallelepiped. The dimensions of each piece are 15cm, 60cm, 40cm. Calculate the total volume in which represents the three pieces.

**Challenge:** Try to draw a regular compound three-dimensional shape on graph paper, composed of three parallelepiped shapes of different volumes, suppose dimensions to them of your choice, then find the volume of the shape and its surface area.

**Open Problem:** How can you estimate the volume of a compound three-dimensional shape of irregular simple three-dimensional shapes?

**Numerical Sense:** A compound three-dimensional shape made of paring a number of wooden cubes whose each edge is 2cm. If its dimensions are 2cm, 4cm, 10cm how many cubes are there?

A mathematical formula for the volume of a compound three-dimensional shape composed of k number of cubes arranged one over the other, their edge length is n cm.
Draw a regular octagon on pieces of colored carton and use the scissors to make a number of different colored octagons. Try to arrange them on top of a table, all together, without leaving spaces between them. Try it again with a regular hexagon. Think why you could arrange the pieces of the regular hexagon while you couldn’t do it with the regular octagon pieces.

**The Regular Polygon**: is a polygon whose dimensions are equal in length and has congruence angles just like the equilateral triangle and the square. The diagonals of the regular polygon are drawn a straight line linking two non-contiguous heads (points).

**Polygon Angle Measurement** = \((\text{the number of its sides } - 2) \times 180^\circ\) divided by the number of sides.

\[
\theta = \frac{(n-2)\times180^\circ}{n}
\]

**Example (1)**

We try to find the measure of each angle in a hexagon as follows: We choose one of its points and starting from it we draw all possible diagonal which are there.

We count the number of triangles formed which are four regular triangles.

Since the sum of triangle angles is \(180^\circ\) so the number of angles in a hexagon is \(4 \times 180^\circ = 720^\circ\).

We calculate the measurement of each angle by dividing the sum of a hexagon’s angles by the number of angles: 
\[720 \div 6 = 120^\circ\]

So the measure of each angle in a hexagon is 120 degrees.
The process of arranging polygons side by side with a specific pattern in such a way that the whole area worked on is covered without any overlapping or left out spaces is called (Paving). In order for the paving process to be done correctly the measure of the angles meeting while paving must be 360 degrees. In order to made a decision about whether a regular polgon is suitable or not for paving if you dive 360 degrees by the measure of the regular polgon angle, the result must be an positive integer and without a remainder. The number of pieces of a regular polygon basically used in paving is determined by dividing the area needed to be paved by the area of the unit used in paving. (a ceramic piece for example) Which usually has constant measurements.

Example (2) Can the floor of a room be paved by using pentagonal pieces of ceramic? Explain why.

1. The pentagonal angle is measured this way:

\[ \theta = \frac{(n-2) \times 180^\circ}{n} = \frac{(5 - 2 ) \times 180^\circ}{5} = \frac{540^\circ}{5} = 108^\circ \]

2. Dive 360° by 108°

\[ \frac{360^\circ}{108^\circ} = 3.3 \]

Where the division result is not an integer, it is not possible to use pentagonal pieces in paving because the measure of the angles meeting in paving is less than 360 degrees.

Example (3) Can the floor of a room be paved by using hexagon pieces of ceramic? Explain why.

1. The hexagon angle is measured this way:

\[ \theta = \frac{(n-2) \times 180^\circ}{n} = \frac{(6 - 2 ) \times 180^\circ}{6} = \frac{720^\circ}{6} = 120^\circ \]

2. Dive 360° by 120°

\[ \frac{360^\circ}{120^\circ} = 3 \]

Where the division result is an integer, it is possible to use hexagon pieces in paving because the measure of the angles meeting in paving is less than 360 degrees.
1. Find the angle measurement of a regular polygon of 12 sides.

2. Can the walls of a kitchen be paved with triangular pieces of ceramic? Explain why.

3. A rectangular wall whose dimensions are 3.6 m, 3 m, should be decorated by being paved with squared pieces of mosaic with a side length of 60 cm. Calculate the number of pieces needed.

4. A squared bathroom floor of side length 2 m is wanted to be paved with rectangular pieces of ceramic whose dimensions are 0.5 cm, 0.25 cm. Calculate the number of pieces needed.

5. A paving worker matches 4 pieces of squared paving cobble the side length of each of them is 25 cm, as shown in the figure. If the area needed to be paved is in the shape of a rectangle with the dimensions 6 cm, 8 cm. Calculate the number of the needed pieces in two ways; the first is by taking the arrangement of the cobble in consider in such a way that the shape is exactly as it is in the figure. The second is without giving importance to that. What is your conclusion?

6. Draw a regular heptagon with all of its possible diagonals. How many triangles would you have?

7. Find the angle measurement for each of the following regular polygons if you knew the number of their sides are:
   - A) 7 sides (heptagon)
   - B) 8 sides (octagon)
   - C) 25 sides

8. Can paving be achieved using the pieces of ceramic described below:
   - A) Regular polygon of 9 sides.
   - B) Regular polygon of 10 sides.
   - C) Regular polygon of 11 sides.

9. If the area of a coloured puzzle piece is 4 cm². How many pieces do we need in order to cover the surface of a rectangular table with the dimensions 30 cm, 40 cm in such a way that the extras in the perimeter are cut and stuck to their suitable places in the blanks left.
Solve the problems

10. A bee cell in the form of a regular hexagon with an area of 25 cm². If the board that carries the cells is rectangular and its dimensions are 40 cm, 60 cm. How many cells does it carry?

11. A house owner wants to surround his rectangular shaped garden (with the dimensions 3 m, 4 m) with squared pieces of coloured ceramic whose surface area is 1 m² in a way that there won’t be any acute edges (using a triangular piece with an area half of that of the piece and a shape of a right triangle) as shown in the figure. It turned out that he needs 16. How much would another house owner need if his garden’s dimensions were 5 m, 6 m?

12. A rectangular wall whose dimensions are 9 m, 18 m contains two windows each in the shape of rectangle with dimensions 1 m, 0.25 m on top of it, half a circle whose area is 0.5 m². It should be paved using pieces of hexagon mosaic, the area of each piece is 2.14 m². Calculate the number of mosaic pieces needed.

Think

13. **Challenge:** Try to find a way where an equilateral triangle can be used in paving if you knew its head corner measure is 70°.

14. **Open Problem:** K is the number of regular polygons. Conclude a rule to find the number of diagonals of the polygon by taking four consecutive values of K.

15. **Numerical Sense:** Without using paper and pencil guess whether the followings in the shape of a regular octagon can be used in paving?

Write

Explain how the parallelogram in the figure can be used in paving the floor of a room?
You have three regular polygons (triangle, square, pentagon).
Count the number of diagonals of any polygon by drawing a straight piece between each two non-contiguous heads in the polygon. Conclude how many diagonals would an heptagon have.

**What are the givens?** The triangle with three sides. The square with four sides. And the pentagon with five sides.

**What is asked?** Counting the number of diagonals for each shape and concluding how many diagonals would a heptagon have.

**How to solve the problem**
Arrange a table with the givens in order to search for the pattern that links the number of sides of the polygon to the number of its diagonals.

<table>
<thead>
<tr>
<th>number of sides</th>
<th>number of diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>No diagonal</td>
</tr>
<tr>
<td>4</td>
<td>2 diagonal</td>
</tr>
<tr>
<td>5</td>
<td>5 diagonal</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7</th>
<th>14</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>number of sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>9</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>number of diagonals</td>
</tr>
</tbody>
</table>

So the number of diagonals of a heptagon (of seven sides) is 14.

**Check**
Make sure your solution is correct by drawing the figure and counting the number of diagonals.
Use “searching for pattern” in order to solve the following problems.

1 **Ceramic:** An engineer used the design as in the figure to make a piece of ceramic used in paving the walls of bathrooms. After he was done, he decided to add another frame to surround the blue one. He used the problem solving strategy (searching for pattern) to know the number of squares of the new frame. Can you guess which colored squares in the corners?

2 **Triangles:** Use the problem solving strategy (searching for pattern) to draw the next two shapes in the polygons’ series.

3 **Overlapping Rectangles:** Use the problem solving strategy (searching for pattern) to find the area of the big rectangle in the figure below if you knew the dimensions of the small rectangle are 2 cm, 4 cm and the dimensions of the rectangles surrounding it increases by 2 each time.
Chapter Review

**Vocabulary**

<table>
<thead>
<tr>
<th>English</th>
<th>Arabic</th>
<th>English</th>
<th>Arabic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduction</td>
<td>تصغير</td>
<td>Compound Planer shapes</td>
<td>أشكال مستوية مرکبة</td>
</tr>
<tr>
<td>Volume</td>
<td>الحجم</td>
<td>Perimeter</td>
<td>المحيط</td>
</tr>
<tr>
<td>Lateral Area</td>
<td>المساحة الجانبية</td>
<td>Area</td>
<td>المساحة</td>
</tr>
<tr>
<td>Total Area</td>
<td>المساحة الكلية</td>
<td>Dilation</td>
<td>التمدد</td>
</tr>
<tr>
<td>Cube</td>
<td>مكعب</td>
<td>Dilation Center</td>
<td>مركز التمدد</td>
</tr>
<tr>
<td>Parallelepipeded</td>
<td>متوازي السطور</td>
<td>Enlargement</td>
<td>تكبير</td>
</tr>
</tbody>
</table>

### [6 - 1] Rate Effect (scale) on Perimeter and Area

**Example 1:** Under the effect of dilation of coefficient 2. Find the image of the square whose head, points are...

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6,2)</td>
<td>(3,1)</td>
</tr>
<tr>
<td>(10,2)</td>
<td>(5,1)</td>
</tr>
<tr>
<td>(10,6)</td>
<td>(5,3)</td>
</tr>
<tr>
<td>(6,6)</td>
<td>(3,3)</td>
</tr>
</tbody>
</table>

**Exercise 1:** Under the effect of dilation of coefficient \( \frac{1}{4} \), find the image of the circle whose center is the origin point and its radius is 16 cm?

**Exercise 2:** In the figure below, if the smaller circle is the image of the big circle under the effect of dilation. Find its coefficient.

### [6 - 2] Calculating Volume

**Example 1:** Find the volume of a parallelepiped whose dimensions are 3 cm, 7 cm, 4 cm

\[
V = L \times w \times h = 3 \times 7 \times 4 = 84 \text{ cm}^3
\]

**Example 2:** A parallelepiped whose base is squared, its height is 12 cm. If its volume is 300 cm³ find its squared base edge length?

\[
V = L \times L \times h \quad 300 = L \times L \times 12 \\
L \times L = \frac{300}{12} = 25 \quad \rightarrow \quad L = 5 \text{ cm}
\]

**Exercise 1:** Find the volume of a parallelepiped whose dimensions are 6 cm, 9 cm, 15 cm.

**Exercise 2:** Find the volume of a cube whose side length is 11cm.

**Exercise 3:** A parallelepiped whose base length is twice its width. And its height is half its width which is 6 cm. Find its volume.
### Example 1:
Find the lateral area and the total area of the parallelepiped whose dimensions are 5 cm, 6 cm, 7 cm.

- L.A. = 2 (L+W) × h
- L.A. = 2 (5+6) × 7 = 154 cm²
- Total Area
- T.A. = 2 (L+w) × h + 2 × L × w
- T.A. = 2 (5+6) × 7 + 2 × 5 × 6
- T.A. = 214 cm²

### Example 2:
Find the lateral and the total area of a cube whose edge length is 5 cm.

- L.A. = 4 × L × L = 4 × 5 × 5 = 100 cm²
- T.A. = 6 × L × L = 6 × 5 × 5 = 150 cm²

### Exercise 1:
Find the lateral area and the total area of the parallelepiped whose dimensions are 4 cm, 7 cm, 10 cm.

### Exercise 2:
Find the lateral and total area of a cube whose edge length is 9 cm.

### Exercise 3:
A parallelepiped of squared base has a height of 10 cm, if its total area is 360 cm², what is the edge length of its base?

### Exercise 4:
A cube whose total area is 216 cm² what is its edge length?

### Exercise 5:
A cube whose edge length is 5 cm was exposed to dilation by coefficient 3. Find its volume and total area after being expanded.

- Total area
- Cube volume before dilation
  - V = (L×L×L) = (5×5×5)
  - V = 125 cm³
- Lateral area before dilation
  - L.A. = 4 × L × L = 4 × 5 × 5
  - L.A. = 100 cm²
- Volume and lateral area after dilation
  - Vₜ = k³ × V = (k.k.k) × V
  = (3×3×3) × 125 = 3375 cm³
  - (L.A)ₜ = k² × (L.A) = (k.k) × (L.A)
  = (3×3) × 256 = 2304 cm²

### Exercise 1:
A parallelepiped whose dimensions are 2 cm, 4 cm, 6 cm. Find its volume and lateral area under the effect of dilation by coefficient 2/3.

### Exercise 2:
If you knew the lateral area of a cube is 64 cm² and the lateral area was reduced under the effect of dilation to become 16 cm². Calculate the amount of dilation coefficient.

### Exercise 3:
A parallelepiped whose base is squared. Its edge length is 4 cm and its height is three times its squared base length. Find its total area under the effect of dilation by \( \frac{3}{4} \).
**Example 1:** 10 similar cubes whose edge is 3 cm were put one over the other. What is the volume of the resulting compound shape?

\[ V = 10 \times (L \times L \times L) \]
\[ V = 10 \times (3 \times 3 \times 3) \]
\[ V = 270 \text{cm}^3 \]

**Example 2:** 3 parallelepipeds were put one over the other. They are similar and have the dimensions 2 cm, 3 cm, 6 cm. Find the volume of the resulting shape.

\[ V = 3 \times (L \times W \times h) \]
\[ V = 3 \times (2 \times 3 \times 6) \]
\[ V = 108 \text{cm}^3 \]

**Exercise 1:** Find the volume of the compound shape consisting of 5 similar cubes put side by side. Each edge of theirs is 4 cm.

**Exercise 2:** Find the lateral area of a compound solid consisting of 4 similar cubes. Each side of theirs is 1.5 cm, and they are placed side by edge.

**Exercise 3:** Find the volume of the compound solid consisting of 4 similar parallelepipeds with the dimensions 2 cm, 3 cm, 6 cm.

---

**Example 2:** Can paving be done if you used a regular cobble of 12 sides?

Here \( n = 12 \) so:

\[ \theta = \frac{(12-2) \times 180^\circ}{12} = 150^\circ \]

**Exercise 1:** Is it possible to pave using a regular cobble of 22 sides?

**Exercise 2:** Find the number of polygons whose side number is 6 and can be drawn in a paving way on rectangular drawing paper with the dimensions 25 cm, 20 cm. If you knew the area for each is 20 cm\(^2\).

**Exercise 3:** Can paving be done using pieces of ceramic. If one pieces had the shape of a trapezoidal, draw an illustrating figure.
Let ABCD be a rectangle whose heads are A(-2,6), B(2,6), C(2,-4), D(-2,-4) draw it and its area, then draw an expansion to it where the center point is at the origing and the coefficient is \( \frac{1}{2} \) and find its area as well.

A cube’s volume is 125 cm\(^3\), find its edge length.

A parallelepiped volume is 96 cm\(^3\), if its base area was 12 cm\(^2\) what is its height?

A parallelepiped with a squared base, its height is 12 cm, if its volume is 864 cm\(^3\), what’s the length of its squared base side?

Find the volume, lateral area, and total area of a cube whose edge is 7cm.

Find the volume, lateral area and total area of a parallelepiped whose dimensions are 15 cm, 10 cm, and 20 cm.

A cube whose edge is 4 cm. Find its original volume and its volume under the effect of dilation with coefficient \( \frac{2}{3} \).

A cube whose side is 4cm. Find its original total area and its area under the effect of expansion with coefficient 4.

If you knew that the total area of a parallelepiped is 17 cm\(^2\), and the total area under the effect of dilation (enlargement) is 153 cm\(^2\), calculate the time it takes to finish melting.

An ice cube whose edge is 20 cm melts, conservative of its geometrical shape, under the effect of dilation coefficient \( \frac{3}{5} \) cm/sec. Calculate the time it takes to finish melting.

A stereophonic compound composed of 8 corresponding cubes, whose each edge is 10 cm, and 4 parallelepiped of similar dimensions 2cm, 4cm, 6cm. Calculate the total volume of the stereophonic solid.

Abdulla wants to pave a squared passage whose side is 9m with a squared cobble. The area of one cobble is 0.25 m\(^2\). Calculate the amount of cobble needed to finish the paving process.
Why the scientists are not able to expect the events of disasters 100%?
How do people use maths when trying to predict or to be ready against disasters?
1. If there were in Muhannad farm 9 trees of apples, 13 trees of fig, 4 trees of almond and 7 trees of palm. Represent these data using signal counting in a table.

Use the data which is given in the table which represents the max. speed of some animals to answer the following questions:

2. Which of the animals is the fastest?
3. Which of the animals whose speed is 40 km/h ?
4. Which of the animals is the slowest, the lion or the rabbit?

<table>
<thead>
<tr>
<th>Animal</th>
<th>km/h Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lion</td>
<td>80</td>
</tr>
<tr>
<td>Rabbit</td>
<td>56</td>
</tr>
<tr>
<td>Elephant</td>
<td>40</td>
</tr>
</tbody>
</table>

Write each of the following fractions as a percentage ratio:

5. \[\frac{70}{100}\]
6. \[\frac{3}{20}\]
7. \[\frac{13}{4}\]
8. \[\frac{6}{5}\]

Arrange the following numbers in ascending order: 22, 15, 30, 20, 31, 11, 18, 20

Using the beside data graph which represents the sport activity of students in one of the school in a specific year to answer the following questions:

10. Which of the activities do the students prefer?
11. Arrange the preferred activities in order.
12. How many students do prefer basketball?

Write each of the following percentage ratio as a decimal form:

13. 13% 14. 7% 15. 20%

Use the given circular data to answer the following questions:

16. What is the percentage ratio of Science subject?
17. What is the angle which represents the Math sector?
18. What is the sector which its percentage ratio is 30%?
Collecting and Organizing Data

Learn

The teacher of Math collected data of twenty weights of students of first intermediate stage, the weights approximated up to kilogram as follows:

\[47, 47, 43, 46, 43, 44, 45, 47, 44, 47, 44, 46, 47, 46, 45, 47, 44, 47, 48, 44\]

i) What is the max. weight among the weights of students?
ii) What is the min. weight among the weights of students?
iii) What is the number of students which weight 47 kg?

Collecting and Organizing Data

To answer the above questions, easily, we need some time to represent them in frequency table, where each value that appears in data will indicate the number of occurrence. It will help me to show data easily.

Example (1)

Arrange the data which the teacher of Math collected into a frequency table:

\[\begin{array}{ccc}
\text{Weights} & \text{Counting Signal} & \text{Frequency} \\
43 & || & 2 \\
44 & |||| | & 5 \\
45 & || | & 2 \\
46 & |||| | & 3 \\
47 & |||| | & 7 \\
48 & | & 1 \\
\end{array}\]
Example (2) Construct a frequency table for the marks of a test. Using the following sets:

<table>
<thead>
<tr>
<th>65</th>
<th>70</th>
<th>83</th>
<th>100</th>
<th>88</th>
<th>78</th>
<th>85</th>
<th>90</th>
<th>50</th>
<th>73</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>63</td>
<td>79</td>
<td>98</td>
<td>86</td>
<td>80</td>
<td>75</td>
<td>55</td>
<td>95</td>
<td>73</td>
</tr>
</tbody>
</table>

i) less than 70, 70-79, 80-89, 90-100.
ii) What is the number of students which they got 80-89?
iii) What is the number of students which they got less than 70?

When the data number is large or the difference between the max. & min. values is large, we construct a frequency table that contains classes to simplify the data expression.

i) The class 70-79 contains all marks of student which lies between 70, 79. The number 70 is called lower limit of the class.
   The number 79 is called the upper limit of the class, and so on for the rest classes.
   • The class 70-79 includes any value of the following:
     70, 73, 76, 77, 79 and the number of elements is 5.
   • Construct a frequency table containing two columns the first contains the classes and the second contains the frequency.
ii) Number of students which have marks 80-89 is 6 students.
iii) Number of students which have marks less than 70 is 5 students.

<table>
<thead>
<tr>
<th>class</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than 70</td>
<td>5</td>
</tr>
<tr>
<td>70-79</td>
<td>5</td>
</tr>
<tr>
<td>80-89</td>
<td>6</td>
</tr>
<tr>
<td>90-100</td>
<td>4</td>
</tr>
</tbody>
</table>

Example (3)

Weather: In one of the months of winter in Baghdad the max. temperatures were recorded in a week as follows:

35, 30, 29, 33, 30, 30, 28
34, 28, 33, 35, 30, 28, 29

i) Represent the data as a frequency table.

ii) What is the max. temperature?

iii) What is the min. temperature?

iv) How many days were the temperature 30°C?

Represent the data a frequency table of the following:

i) Arrange the data in ascending and evaluate the frequency of each value.

ii) Max temperature was 35 °C

iii) Min temperature was 28 °C

iv) The number of days which were the temperature 30°C was 4 days.
Make sure of your understanding

1. Arrange the following data in a frequency table:
   2, 2, 1, 6, 2, 3, 5, 5, 1, 2, 6, 5, 4, 6, 1, 3, 1

   **Measurement:** The following represents the heights of 16 first class intermediate students in cm. as follows:
   150, 137, 149, 136, 146, 148, 145, 134
   149, 135, 137, 144, 136, 131, 141, 138

   **Construct a frequency table of height of students using the following sets:**

   2. Less than 140, 140-145, 146-150.

   3. What is the number of students which their heights are less than 140 cm?

   4. What is the number of students which their heights are between 140, 150 cm?

---

Solve the Exercises

5. Arrange the following data in a frequency table
   2, 3, 1, 4, 3, 3, 6, 5, 1, 2, 4, 6, 1, 3, 2, 5

   A scientist researched on 10 volunteers their sleeping time and concluded the outcomes:
   365, 435, 380, 460, 400, 425, 440, 490, 500, 505

6. Use the results to construct the frequency table using the following sets:
   Less than 400, 400-450, 451-505

7. What is the number of volunteers which is less than 400?

8. What is the number of volunteers which is within 451-505?
Solve the problems

9 **Workers:** The following data represent the ages of few workers in one of the companies:
   - 53, 52, 45, 29, 45, 27
   - 27, 31, 50, 32, 30, 47

Construct a frequency table for the given above data.

10 What is the number of the youngest age of workers and what is the number of the oldest age?

11 What is the number of workers whose each age is less than 30 years?

12 What age is recorded most than others among the workers in the company?

**Health:** The following data represent the weights of twenty babies in kilogram
   - 30, 16, 20, 30, 26, 22, 33, 19, 24, 15
   - 32, 26, 18, 27, 31, 25, 32, 23, 21, 19

13 Construct a frequency table using the following sets:

14 What is the number of babies their weights are less than 30 kg.?

15 What is the number of babies their weights are between 30-34 kg.?

Think

**Challenge:** If the number of goals that been attained in football games were as follows:
   - 4, 5, 3, 2, 2, 2, 1, 0, 1, 6, 4, 5, 2

16 Look at those data ascending from 0-6, taken three equal times, and then construct frequency table.

17 Can we display the same data by using the frequency table and signal of counting? Explain your answer.

Write

A problem from the life fact which can be solved by frequency table.
Circular Sectors

Idea of the lesson
Representation of data by means of circular sectors and to explain them.

Vocabulary
Graph circle
Sector

Learn

One of the students in college of Agriculture reconnoitered among animals breeding in countryside homes, so he found that 75% they breed chickens, 15% breed dogs and 10% breed cats. How does the student show his conclusion of recon number.

Circle Graph

Circle graph is the best choice to represent the percentage, because it shows clearly the area of the sectors and the comparison among them.

Example (1)
Find the angle value of each sector.

Step 1:
- Angle value of chickens sector: \( \frac{75}{100} \times 360° = 270° \)
- Angle value of dogs sector: \( \frac{15}{100} \times 360° = 54° \)
- Angle value of cats sector: \( \frac{10}{100} \times 360° = 36° \)

Step 2:
We draw the circle graph by using a compass, protractor and ruler, by starting from a certain radius, to draw the sector of each sector.

Step 3:
Give a name to each sector and colour it.

Example (2)

Agriculture: A farm contains 200 trees the given circle graph shows the percentage of each species. Find the number of each species of trees, then answer the following:

i) Which of the species of trees has a least number of trees?

ii) Which of the species of trees has a number greater than half of the whole number of trees in the farm.

iii) Which of species has a percentage 25%?

Number of orange trees: \( \frac{60}{100} \times 200 = 120 \)
Example (3) The table beside shows the results of favourite colours by some students. Draw a circle graph which represents the given data in the table.

**Step 1:** Find the sum of the number of the students

\[ 6 + 15 + 9 = 30 \]

**Step 2:** Find the angle of each sector:

- Sector angle of blue coloured sector: \( \frac{6}{30} \times 360^\circ = 72^\circ \)
- Sector angle of red coloured sector: \( \frac{15}{30} \times 360^\circ = 180^\circ \)
- Sector angle of green coloured sector: \( \frac{9}{30} \times 360^\circ = 108^\circ \)

**Step 3:** Draw the circle graph and name of each sector and colour it.

Example (4) Survey: The following table shows the opinions of 40 persons dealing with their favourite hobbies.

<table>
<thead>
<tr>
<th>favourite hobbies</th>
<th>percentage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>hobby</td>
</tr>
<tr>
<td>reading</td>
<td>15%</td>
</tr>
<tr>
<td>computer games</td>
<td>5%</td>
</tr>
<tr>
<td>walking</td>
<td>55%</td>
</tr>
<tr>
<td>swimming</td>
<td>25%</td>
</tr>
</tbody>
</table>

i) How many persons do they prefer walking?

ii) What is the sector angle of swimming sector?

Number of persons who prefer walking:

\[ \frac{55}{100} \times 40 = 22 \]

The angle of swimming sector:

\[ \frac{25}{100} \times 360^\circ = 90^\circ \]
Make sure of your understanding

Use the given table and draw the circle graph.

1. Which of the discs represents an angle 90°?
2. Which of the discs has a ratio 30%?

<table>
<thead>
<tr>
<th>Set of compact disc of muhammad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Educational programmers</td>
</tr>
<tr>
<td>language dictionary</td>
</tr>
<tr>
<td>Art programmes</td>
</tr>
<tr>
<td>Luxury games</td>
</tr>
</tbody>
</table>

If Muhammad had 20 compact disc (CD) use the table in question 1 and find the number of CD of each species and then answer the following:

3. Which of the CD is the least in number?
4. Are the luxury game discs greatest in number?
5. What is the number of discs which represent the luxury games?
6. What is the value of angle which represents the luxury games sector?

Solve the Exercises

Newspaper: The given table shows the newspapers that a group of persons prefer.

7. Represent the data of the table in a circle graph, and then answer the followings:
8. Which of the newspapers has a ratio of 35%?
9. What is the angle of the advertisement news sector?

<table>
<thead>
<tr>
<th>Preferred News</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>News</td>
<td>7</td>
</tr>
<tr>
<td>Sport</td>
<td>3</td>
</tr>
<tr>
<td>Advertisement</td>
<td>5</td>
</tr>
<tr>
<td>Economic</td>
<td>4</td>
</tr>
<tr>
<td>Other</td>
<td>1</td>
</tr>
</tbody>
</table>

Time: A student spends 10 hours with his family as shown in the table below.

10. Represent the data as circle graph.

<table>
<thead>
<tr>
<th>The time of which the student spends with his family</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>food</td>
<td>35%</td>
</tr>
<tr>
<td>watching TV</td>
<td>25%</td>
</tr>
<tr>
<td>Conversation</td>
<td>15%</td>
</tr>
<tr>
<td>Sport</td>
<td>25%</td>
</tr>
</tbody>
</table>

Answer the followings:

11. Which of the times has the same percentage?
12. Which of the time has an angle of sector 54°?
13. What is the angle of sport sector?
14. How many hours does the student spend his time in conversation?
Weather: The following table shows the monthly average of temperature in one of the resorts of Iraqi Kurdistan.

Represent the given data as a circle graph and answer the followings:

<table>
<thead>
<tr>
<th>Month</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>February</td>
<td>10</td>
</tr>
<tr>
<td>March</td>
<td>19</td>
</tr>
<tr>
<td>April</td>
<td>15.5</td>
</tr>
<tr>
<td>May</td>
<td>22.5</td>
</tr>
<tr>
<td>June</td>
<td>23</td>
</tr>
</tbody>
</table>

16 What is the percentage of temperature in May?
17 Which of the months has an angle its sector 92°?
18 What is the angle of sector February?
19 Which month has a percentage equals 25%?

Challenge: Some of people which have swimming hobby, prefer walking instead of swimming. Accordingly the value of their sector angle became 216°. What is the number of those people?

Use table in Example 4

The approximate area of the land of the earth is 150 million square kilometers. The area of South Pole Continent is 10% of the area of the land. Find the area of the South Pole continent approximately.

A problem from your life fact that can be solved by using circle graph.
The data below in F.T. represents the interest of one of mini-markets approximated to thousand dinars within thirty days.

How can I help the dealer to extract a new information from the given original data of the table?

**Frequency Polygons**

**Frequency Polygons**: It is one of the methods to extract new informations about the original data. It is a number of segments which join the points whose components are the center of classes and the frequency.

**Center of class**: It is the sum of lower limit upper limit of the class divided by 2.

We usually join the ends of the polygon with x-axis, so we join the beginning with a certain center of classes which lies on the left of the first class and its frequency is zero, while the end of the polygon joins the mid-mark which lies on the right of the last class on x-axis, and its frequency equals zero too.

### Example (1)

Represent the data as a paragraph (learn) by frequency polygons.

**Step 1**: Construct the following table:

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Class of interest</th>
<th>center of classes</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6 - 10</td>
<td>$\frac{6 + 10}{2} = \frac{16}{2} = 8$</td>
<td>(8 , 5)</td>
</tr>
<tr>
<td>11</td>
<td>11 - 15</td>
<td>$\frac{11 + 15}{2} = \frac{26}{2} = 13$</td>
<td>(13 , 11)</td>
</tr>
<tr>
<td>8</td>
<td>16 - 20</td>
<td>$\frac{16 + 20}{2} = \frac{36}{2} = 18$</td>
<td>(18 , 8)</td>
</tr>
<tr>
<td>6</td>
<td>21 - 25</td>
<td>$\frac{21 + 25}{2} = \frac{46}{2} = 23$</td>
<td>(23 , 6)</td>
</tr>
</tbody>
</table>
**Step 2:** Draw two perpendicular gradated coordinates which the x-axis represents center of classes and the y-axis represents the frequency, and then determine the points.

**Step 3:** We join those points by segments using a ruler, to get the given figure.

**Step 4:** The polygon description: We see through the frequency polygon that the interest increases up to first 15 days and then it decreases after 13 days.

---

**Example (2)**

Represent the given data by frequency polygon and describe the changes that occur to data.

- We construct the following table using the given data in the given example:

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Classes</th>
<th>center of classes</th>
<th>Points</th>
<th>Frequency</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15 - 19</td>
<td>( \frac{15 + 19}{2} = \frac{34}{2} = 17 )</td>
<td>(17, 2)</td>
<td>2</td>
<td>15 - 19</td>
</tr>
<tr>
<td>4</td>
<td>20 - 24</td>
<td>( \frac{20 + 24}{2} = \frac{44}{2} = 22 )</td>
<td>(22, 4)</td>
<td>4</td>
<td>20 - 24</td>
</tr>
<tr>
<td>8</td>
<td>25 - 29</td>
<td>( \frac{25 + 29}{2} = \frac{54}{2} = 27 )</td>
<td>(27, 8)</td>
<td>8</td>
<td>25 - 29</td>
</tr>
<tr>
<td>6</td>
<td>30 - 34</td>
<td>( \frac{30 + 34}{2} = \frac{64}{2} = 32 )</td>
<td>(32, 6)</td>
<td>6</td>
<td>30 - 34</td>
</tr>
</tbody>
</table>

- Describe the data from the above table. It is clear from the frequency polygon that the data increases up to 25 and starts to descend.
1. Represent the data frequency table by frequency polygon and describe the changes which occur to the data:

<table>
<thead>
<tr>
<th>Class</th>
<th>5 - 9</th>
<th>10 - 14</th>
<th>15 - 19</th>
<th>20 - 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td>7</td>
</tr>
</tbody>
</table>

**Health:** The following table indicates the weights of twenty babies in kilograms.

<table>
<thead>
<tr>
<th>Class of weights</th>
<th>15 - 21</th>
<th>22 - 28</th>
<th>29 - 35</th>
<th>36 - 43</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

2. Represent these data by frequency polygons.
3. Describe the changes which occur to these data.

**Solve the Exercises**

**Sport:** The following table shows the results of twenty sports men in high jumping game.

<table>
<thead>
<tr>
<th>High jumping</th>
<th>11 - 20</th>
<th>21 - 30</th>
<th>31 - 40</th>
<th>41 - 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

4. Represent these data by frequency polygon.
5. Describe the changes that occur to these data.

6. Represent the data of frequency table by frequency polygon and describe the changes which occur to these data.

<table>
<thead>
<tr>
<th>Classes</th>
<th>21 - 25</th>
<th>26 - 30</th>
<th>31 - 35</th>
<th>36 - 40</th>
<th>41 - 45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>11</td>
<td>16</td>
<td>19</td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>
Solve the problems

**Learn:** The following data represent the number of first class intermediate students which they memorized Surats of Kuran Karim. They are classified into classes which represent the number of surat.

<table>
<thead>
<tr>
<th>Classes</th>
<th>6 - 10</th>
<th>11 - 15</th>
<th>16 - 20</th>
<th>21 - 25</th>
<th>26 - 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>6</td>
<td>7</td>
<td>12</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Represent these data by frequency polygon.

Describe the changes that occur to those data.

**Think**

**Challenge:** Number of pages of a chapter from a story is (35) pages, they are divided equally by 5 groups to read, number of elements of groups are 6, 2, 5, 3, 4 respectively.

- Represent these data by frequency table.
- Represent these data by frequency polygon.

The given bar diagram represent the distances of some students’ houses from their school in meters.

**Basketball:** The frequency table shows (28) sportsmen playing a basketball.

<table>
<thead>
<tr>
<th>Classes</th>
<th>10 - 20</th>
<th>21 - 31</th>
<th>32 - 42</th>
<th>43 - 53</th>
<th>54 - 64</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of sportsmen</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

Represent the data as frequency polygon.

**Write**

The steps of data representation by a frequency polygon.
Idea of the lesson:
- Representing the data by stem and leaf.

Vocabulary
- Stem
- Leaf

One of the specialists studied gathering data of weights of (20) students in intermediate stage, approximated to kilogram. The results were as the following:

39 42 42 40 38 39 42 41 38 42
42 43 39 42 40 40 41 41 42 39

i) What is the maximum weight among these students?
ii) What is number of students that weigh each 38 kg.

Stem and Leaf

Representation by stem and leaf, arranging the data from smallest to biggest. The leaves represent the unit which has low value, while the stem represents the digits in the greatest common place value.

Example (1) Use the representation of stem and leaves to show the data.

Step 1: Note that the data consists of two digits, so the tens place represents the stem and the ones place represents the leaves. Choose the stems by using the number 3,4 (tens place).

Step 2: Make the following table which consists two perpendiculars, stem and leaves.
- Write the stems on their perpendicular from smallest to largest. (tens)
- Write the leaves of every stem on the right even if they are repeated. (ones)

<table>
<thead>
<tr>
<th>tens place</th>
<th>Leaves (ones place)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9 8 8 9 9</td>
</tr>
<tr>
<td>4</td>
<td>2 2 0 2 1 2 2 0 0 1 1 2</td>
</tr>
</tbody>
</table>

Step 3: Arrange the leaves from smallest to biggest, even if they are repeated.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8 8 9 9 9</td>
</tr>
<tr>
<td>4</td>
<td>0 0 0 1 1 2 2 2 2 2 2 2 2</td>
</tr>
</tbody>
</table>

78
**Step 4:** By using the table answer the questions.

Note that the stem 4 has the most number of leaves which contains 13 leaves. The leaf 2 in the stem repeated 7 times, so the weights 42 was the largest weights.

Note the stem 3 has 5 leaves, leaf 8 in stem 3 repeated twice, therefore the number of the students which their weights 38 kg. is two only.

**Example (2)** The following table shows the height of some trees in a farm in meters.

i) Use the table to represent the data by stem and leaves.
ii) What is the number of trees which their height less than 18 m?
iii) What is the number of trees which their height is 20 m?

<table>
<thead>
<tr>
<th>Height of trees in meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
</tr>
<tr>
<td>11</td>
</tr>
</tbody>
</table>

Use the representation of stem and leaves to show the data.
• Number of trees which their heights less than 18 m is 5 trees.
• The leaf 0 in stem 2 repeated twice.
  So the number of trees which their height 20m is 2 trees.

**Example (3)**

**Speed:** The below table shows the speed of cars in kilometer per hour on a road.

i) Represent the data by stem and leaf.
ii) What is the number of cars which their speed is 74 km/hr?
iii) What is the number of cars which their speed is greater than 65 km/h?
iv) What is the min. speed among these cars?

**i) Answer:**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>5 5 5 8 8 9 9</td>
</tr>
<tr>
<td>7</td>
<td>0 1 1 2 4 4 6</td>
</tr>
</tbody>
</table>

- Number of cars which their speed 74 km/h is 2.
- Number of cars which their speed is greater than 65 km/h is 11 cars.
- The minimum speed among the cars is 59 km/h.
Make sure of your understanding

1. **Students:** Use the stem and leaves representation to display the data in the given table

<table>
<thead>
<tr>
<th>Students’ Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>91 96 80 91</td>
</tr>
<tr>
<td>93 79 93 80</td>
</tr>
<tr>
<td>72 80 93 70</td>
</tr>
</tbody>
</table>

2. What is the number of students whose marks are less than or equal 80?

   **Weather:** The max temperature in °C as follows:
   
   15 13 28 32 38 30 13 36 35 20 24 38 32 38

3. Use the stem and leaves representation to display the above data.

4. Number of temperatures which are less than 30 °C.

5. How many days their temperature were greater than 32 °C?

Solve the Exercises

**Books:** The following table shows the prices of books

<table>
<thead>
<tr>
<th>Price of books</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 16 19 30</td>
</tr>
<tr>
<td>31 29 25 25</td>
</tr>
<tr>
<td>28 39 34 35</td>
</tr>
</tbody>
</table>

6. Use the stem and leaves representation to display (show) the data in above table.

7. How many books are there, where the price of each is less than 25 thousand dinars?

8. How many books are there the which the price each of them between 25 and 39 thousand dinars?

9. Use the stem and leaves representation to display data in the below table.

<table>
<thead>
<tr>
<th>Number of pages of chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 9 15 21</td>
</tr>
<tr>
<td>8 18 17 10</td>
</tr>
<tr>
<td>22 20 11 12</td>
</tr>
</tbody>
</table>

10. How many chapters are there which their each contains number of pages greater than 18?

11. How many chapters are there which their each one has 10 pages?
Solve the problems

**Birds:** The following data show the average weights of small birds among 12 different species in kilogram.

13 11 6 19 7 10
12 5 12 21 18 25

12 Use the stem and leaves representation to display these data.
13 Which of these birds have the lightest weight?
14 How many weights of birds are less than 12 kg?
15 Use the stem and leaves representation to show the data in the given table which represents the number of visitors of a library within a certain time.

16 What is the number of visitors, each of them has more than 20 visiting?
17 How many visitors of a library each of them has visiting 17 times?

**Think**

The following represents marks of the first class intermediate students in math are represented by stem and leaves, as it is shown in below table:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3 7</td>
</tr>
<tr>
<td>5</td>
<td>3 6 7 8</td>
</tr>
<tr>
<td>6</td>
<td>0 0 3 5 7 8</td>
</tr>
</tbody>
</table>

Use the table and answer the followings:

18 Which of the marks were most repeated among the marks of students?
19 How many students failed in math?
20 How many students did get less than 60 marks?

**Sport:** The following data represent the winning number that made by ten countries in bicycle racing.

1 4 10 18 36 19 12 12 11 5

21 Use stem and leaves representation to show data and answer the following:

22 How many countries that won more than 10 times?
23 How many countries that won 12 times in the race?

**Write**

A problem from life fact and then represent by stem and leaves.
Outcomes of Experiment and Organizing

**Experiment:** The is an action to observe the outcomes. The outcomes can be arranged by two methods, table and tree, and these outcomes are written inside two brackets of type \( \{ \} \), this set is called “Sample Space”.

**Example (1)**

**First method:** Tree graph
The sets can be created by a paragraph (Learn) by tree graph method.

```
cube

/ \  
red blue

/ \  
parallelepiped

/ \  
red blue
```

Sample Shape Colour Outcome
--- --- --- ---
cube red Red cube
parallelepiped blue Blue parallelepiped
cube blue Blue cube
parallelepiped red Red parallelepiped

**Second method:** Table
The sets can be created by a paragraph (Learn) by means of table method.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Shape</th>
<th>Colour</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>🟠</td>
<td>cube</td>
<td>red</td>
<td>Red cube</td>
</tr>
<tr>
<td>🔵</td>
<td>parallelepiped</td>
<td>blue</td>
<td>Blue parallelepiped</td>
</tr>
<tr>
<td>🔵</td>
<td>cube</td>
<td>blue</td>
<td>Blue cube</td>
</tr>
<tr>
<td>🟠</td>
<td>parallelepiped</td>
<td>red</td>
<td>Red parallelepiped</td>
</tr>
</tbody>
</table>
Food: If you want to eat a sandwich of chicken or a sandwich of meat with cheese or tomato or potato.
Find the outcomes using:
i) Tree graph   ii) Table

First method: Tree graph

Number of equals six
Sample Space is \{ (Chicken,Cheese), (Chicken, Tomato), (Chicken, Potato),
(Meat,Cheese), (Meat, Tomato), (Meat, Potato)\}

Second Method: Table

<table>
<thead>
<tr>
<th>Sandwich</th>
<th>Species</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicken</td>
<td>Cheese</td>
<td>(Chicken,Cheese)</td>
</tr>
<tr>
<td>Meat</td>
<td>Tomato</td>
<td>(Meat, Tomato)</td>
</tr>
<tr>
<td>Chicken</td>
<td>Potato</td>
<td>(Chicken, Potato)</td>
</tr>
<tr>
<td>Meat</td>
<td>Cheese</td>
<td>(Meat, Cheese)</td>
</tr>
<tr>
<td>Chicken</td>
<td>Tomato</td>
<td>(Chicken, Tomato)</td>
</tr>
<tr>
<td>Meat</td>
<td>Potato</td>
<td>(Meat, Potato)</td>
</tr>
</tbody>
</table>

Example (3)

The pointer of the revolving dics is rotated twice. Write all the probabilities using the tree method.

Number of probabilities equals 9

Sample Space

1 **Playing:** In a park there are a swing, jump playing and hanged-ladder playing. In how many different ways you can perform all of them in playground? Represent the outcomes using tree graph.

Tossed a coin twice, answer the followings:

2 Represent all the outcomes by using a table:

3 What is the number of different sets of possible probability?

---

**Fashion:** Maha has a pair of black, white and brown shoes and black skirt and brown skirt.

4 In how many different ways she can dress? Represent the outcomes using a table.

The below table shows the names of geometrical figures with two colors.

<table>
<thead>
<tr>
<th>Figure</th>
<th>circle, square, triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colour</td>
<td>yellow, blue</td>
</tr>
</tbody>
</table>

5 Use the tree graph to construct all possible probability sets.

6 **Letters:** Choose letters from the word “Kirkuk”, corresponded with letters of “Cat” use the tree method and write all possible probability.

7 **Food:** If you have four choices of main food plate three choices of fruits and two choices of sweets. What is the number of different meals can be chosen from these choices?

8 What kind of information that tree graph can be provided you?

9 Is the tree graph beneficial to determine the outcomes of tossing a coin? Explain your answer.
Three different coins are tossed. Use the tree graph method to represent all possible probability.

You have three different volume of cubes, small, medium and large with two colours red and blue. Use the table to represent all possible probabilities.

Open problem: If possible outcome of tossing coins once all together as follows: 

(H, H, T), (H, T, H), (H, T, T),
(T, H, H), (T, H, T), (T, T, H),
(T, T, T)

How many coins were tossed?

By using the given tree representation (graph) complete the missing probability.

Number of possible probability equals

If the number of probabilities of tree representation by certain data equals six. Choose the data which satisfy it.

How many tree representations give these outcomes? Represent them by letters and numbers.

The steps which you follow to make tree graph that show your experiment outcomes.
Comparing between Probabilities

Experiment: It is an action leads to number of possible outcomes each of them is called “Event”, which is a set containing one outcome or more.

Event Probability: It is a measurement of eventual chance.

Certain event: The event that happens certainly.

Impossible event: The event that does not happen at all.

Probable event: The event that has a chance to happen. If it has a large chance then it is called “More likely event” and if it has a small chance then it is called “Less likely event”.

Example (1)

i) The probability of drawing red ball from the first box is a certain probability because all the balls in first box are red.

ii) Probability of drawing yellow ball from the first box drawn by Muhannad is an impossible probability because there is no yellow ball in the first box at all.

iii) Probability of red ball drawn from the second box is more likely probable because the number of red balls is the most among the others.

iv) Probability of green balls drawn from the second box is less likely because the number of green balls less than the number of red balls in the box.

Muhannad has two boxes, first box contains all red balls and the second box contains different color balls. If Muhannad has drawn one ball from each box, what is the probability of red ball?
Example (2)
A box contains 3 cubes, 3 spheres and 5 cones.

i) Which event has greater probability drawing sphere or cone?
Since the number of cones is greater than the number of spheres, so the probability of drawing a cone is greater than the probability of drawing a sphere.

ii) Which event has greater probability drawing sphere or cube?
Since the number of spheres equals the number of cubes, therefore their events have equal probability.

Example (3)  The probability can be written as a percentage ratio write the probability of each event.

Write the probability of each event:
Write 0% or 50% or less than 50% or greater than 50% or 100%

i) All female students in first-class intermediate school are present today. Maryam is a student in first class.
What is the probability of Maryam being present today?
The probability is certain, therefore she is present in the school 100%.

ii) An airplane of Iraqi Airways contains a number of passengers, but Muhammad was not in it after taking-off.
What is the probability that he was in the airplane after taking-off?
The probability is impossible, therefore Muhammad’s presence probability in the airplane is 0%.

iii) What is the probability of appearing 6 when a dice is tossed?
The probability is less likely, therefore the probability is less than 50%.

iv) A sack contains 10 red balls and 10 yellow balls.
What is the probability of drawing a red ball?
The probability is likely equal, therefore the probability of drawing a red ball is 50%.

v) A box contains 7 cubes and 3 spheres. What is the probability of drawing a cube?
The probability is more likely, therefore the probability is greater than 50%.

Example (4)
The pointer of a revolving disc is rotated once. Write probability for the following:

i) The pointer stops on red colour. More likely greater than 50%

ii) The pointer stops on green colour. Less likely less than 50%

iii) The pointer stops on black colour. Impossible probability, 0%.
Make sure of your understanding

Write the probability of each event: certain, impossible, probable, more likely and less likely.

1. The number 13 is an odd number ............
2. \(3 \times 6 = 18\) ..............................
3. \(18 = 3 \times 6\) ..............................
4. \(5^3 = 5 \times 5\) ..............................

The disc is rotated once, answer the following:

5. The pointer stops on green colour. ....................... ..............................
6. The pointer stops on yellow colour. ..............................
7. The pointer stops on blue colour. ..............................
   Write the probability of every event. Write 0% or 50% or less than 50% or 100%.

8. The probability of coming Tuesday after Mondays is ..............................
9. The probability of drawing a yellow ball from a box containing all red balls is ..............................
10. The probability of appearance of number 5 when tossing a dice is ..............................
11. A committee consists of 3 men and 3 women. The probability of choosing one man is ..............................

Solve the Exercises

Inscribed the right answer and write the suitable word (certain, impossible and probable)

12. **Weather:** The temperature of one of summer days are 2 °C, 39 °C.
13. **Numbers:** The number 0 is odd, even
14. **Gardens:** In a small garden there are 7 plants of red rose, 3 plants of white rose. If a plant randomly chosen, what is the greatest probability of the plant that has a red colour or white colour? Prove your answer.
   Write the probability of each event. Write 0% or %50 or less than 50% or 100%.
15. Number of days in a month is 30 days. ..............
16. Baghdad is the capital of Iraq Republic ..............
17. The number 5 is one of the factors of 12 ..............
   a sac contains 3 yellow balls and 3 green balls.
18. Drawing a yellow ball ..........
**Play:** Suppose you turned the given disc once. Describe probability of pointer’s stop on every colour. Write (certain, more likely, less likely, impossible)

19. Green but not red ......................
20. Blue ......................
21. Red but not green ......................
22. Not brown ......................

Write: probability of every event, write (0%, 50%, less than 5% greater than 50% or 100% ).

**Cards:** Khalid pull one card among the following cards randomly.

23. The letter A ..............
24. The letter B ..............
25. The letter C ..............
26. The letter A or D ..............
27. The letter H ..............

**Think**

**Challenge:** Describe a set of 7 balls with following characteristics:

28. There are 4 different colours in the set.
29. When a ball is drawn the probability of drawing a coloured ball is greater than the probability of any other coloured ball
30. Only two among other colours have equal probable.
31. You have a cube 3 and parallelepiped 4 which event whose probability is greater than the number 3 appearance on the cube or on the parallelepiped? Describe your answer.

**Write**

A problem from life fact that can be solved by describing the probability, then solve that problem.
Muhammad has 25 pieces of triangles and circles. If each 3 triangles corresponds two circles. How many triangle does Muhammad have?

Understand

**What is the given of the problem?**

Set of triangles and circles their total number is 25 and each 3 triangles corresponds 2 circles.

**What do we want to find?**

The number of triangles does Muhammad have.

Plan

**How can you solve the problem?**

Make a model to every problem.

Solve

I use the pieces such that each 3 triangles correspond 2 circles and as follows:

I got 5 sets in each of set there are 3 triangles and 2 circles.

Add those number of triangles, so we get 15.

Therefore the number of triangles equals 15 triangles.

Check

We have 15 pieces of triangles and 10 pieces of circles.

Add : $15 + 10 = 25$

Therefore the answer is correct.
Problems

1. How many pieces of soap can be put in a box widens 7 layers of soap, if the first layer contains 10 pieces?

2. The distance around a circular pitch is 50 m., if each person stopped a distance 5 m. far from another person. How many persons are there in the pitch?

3. Tariq wants to arrange 18 square-shaped flagstones in a rectangular shape, provided that the perimeter must be least as possible. How many flagstones will be paved on each row?

4. Four students raced. If Ahmad raced to the final after Khalid and before Muhammad, and Saad raced to the final after Ahmad and before Muhammad. Who is the winner in the competition?

5. A box contains 30 red and green balls, if every four red balls corresponds two green balls. What is the number of green balls in the box?
## Chapter Review

### Vocabulary

<table>
<thead>
<tr>
<th>English</th>
<th>Arabic</th>
<th>English</th>
<th>Arabic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Certain Event</td>
<td>احداثة. أتآكد</td>
<td>Frequency Table</td>
<td>الجدول التكراري</td>
</tr>
<tr>
<td>Experimen</td>
<td>تجربة</td>
<td>Class</td>
<td>فئة</td>
</tr>
<tr>
<td>Stem</td>
<td>ساق</td>
<td>Frequency</td>
<td>تكرار</td>
</tr>
<tr>
<td>Leaf</td>
<td>ورقة</td>
<td>Circular Sector</td>
<td>قطاع دائري</td>
</tr>
<tr>
<td>Probable</td>
<td>ممكن</td>
<td>Frequency Polyon</td>
<td>قطاع تكراري</td>
</tr>
<tr>
<td>More likely</td>
<td>ممكن قوي</td>
<td>Outcome</td>
<td>النتائج</td>
</tr>
<tr>
<td>Less likely</td>
<td>ممكن ضعيف</td>
<td>Tree</td>
<td>الشجرة</td>
</tr>
<tr>
<td>Circle Graph</td>
<td>الدائرة البيانية</td>
<td>Probability</td>
<td>الأحتمالية</td>
</tr>
<tr>
<td>Center of Class</td>
<td>مركز الفئة</td>
<td>Event</td>
<td>الحدث</td>
</tr>
<tr>
<td>Sector</td>
<td>قطاع</td>
<td>Impossible Event</td>
<td>الحدث المستحيل</td>
</tr>
</tbody>
</table>

### [7 - 1] Collecting and Organizing Data (frequency table)

**Example:** The ages of outpatients visitors to one of the clinical center are given below:

1  11  15  3  50  40  10  60  
51  35  7  12  44  55  30

Represent the data as frequency table

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Counting Signal</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 10</td>
<td>III</td>
<td>4</td>
</tr>
<tr>
<td>11 - 20</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>21 - 30</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>31 - 40</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>41 - 50</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>51 - 60</td>
<td>III</td>
<td>3</td>
</tr>
</tbody>
</table>

*What is the age class which most visited the center? 1 - 10*

*What is the number of outpatients which their ages exceed 40 years? 5 visitors*

*What is the number of outpatients which their age of each of them 30 years old? 1 Person*

**Exercise:** A fair has a set of second-hand cars. The production year are given below:

95  94  81  99  97  85  
79  76  83  93  86  91  
77  78  87  88  75  82

Represent the data in frequency table using the sets:

70 - 79 , 80 - 89 , 90 - 99

*What is the number of cars which the production year exceed 85?*

*What is the number of cars which their production year 81?*

*What is the number of cars which their production year varies within 90 - 99?*
[7 - 2] Circular Sectors

Example: In the library of school there are 300 books, 50% syllabus, 20% literature and 30% miscellaneous. Compute the number of books of each kind in the library.

\[
\frac{50}{100} \times 300 = 150 : \text{number of syllabus books} \\
\frac{20}{100} \times 300 = 60 : \text{number of literature books} \\
\frac{30}{100} \times 300 = 90 : \text{number of miscellaneous books.}
\]

Exercise 1: Represent the given data beside by circular sectors.

Exercise 2: The circular graph represents the favourite juice of 40 persons.
* Which of the juice is the most favourite?
* What is the number of persons who prefer the orange juice?
* What is the value of sector angle of melon juice?

[7 - 3] Frequency Polygons

Example: The table below shows the ages of three sets of men. Represent these data by frequency table.

<table>
<thead>
<tr>
<th>Age class</th>
<th>21 - 25</th>
<th>26 - 30</th>
<th>31 - 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>11</td>
<td>16</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>frequency</th>
<th>class</th>
<th>center of class</th>
<th>points</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>21 - 25</td>
<td>( \frac{21 + 25}{2} = \frac{46}{2} = 23 ) ( (23,11) )</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>26 - 30</td>
<td>( \frac{26 + 30}{2} = \frac{56}{2} = 28 ) ( (28,16) )</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>31 - 35</td>
<td>( \frac{31 + 35}{2} = \frac{66}{2} = 33 ) ( (33,5) )</td>
<td></td>
</tr>
</tbody>
</table>

Exercise: By using the given table of data representation to construct a frequency polygon

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 - 10</td>
<td>5</td>
</tr>
<tr>
<td>11 - 15</td>
<td>11</td>
</tr>
<tr>
<td>16 - 20</td>
<td>8</td>
</tr>
<tr>
<td>21 - 25</td>
<td>6</td>
</tr>
</tbody>
</table>
[7 - 4] Stem and Leaf

Example: The following data represents the ages of a number of workers in one of the companies.
27 35 38 45 48 36
23 31 29 40 45 52
53 45 52 29 50 31

Represent the data by stem and leaves and answer the following:
* What is the youngest and oldest workers in the company?
* What is the age-class which has the most frequency (repetition)?

Exercise 1: The following data represent the weights of a number of students in kg.:
35 37 28 41 36 28
31 52 51 44 47 33

Represent the data by stem and leaves.
* What is the number of students which their weights are less than 40 kg.?
* What is the greatest and least weight among the students?

Exercise 2: Write all the data which are represented in the following table:

<table>
<thead>
<tr>
<th>stem</th>
<th>leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3 3 5</td>
</tr>
<tr>
<td>2</td>
<td>4 1</td>
</tr>
<tr>
<td>5</td>
<td>0 0 0 3 8</td>
</tr>
</tbody>
</table>

* The youngest workers is 23 years old.
* The oldest workers is 52 years old.
* The class 30 years.

[7 - 5] Outcomes of Experiment and Organizing

Example: Two coins are tossed once together. The outcomes of the experiment are represented in the following table.

<table>
<thead>
<tr>
<th>First Coin</th>
<th>Second Coin</th>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>(H, H)</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>(H, T)</td>
</tr>
<tr>
<td>T</td>
<td>H</td>
<td>(T, H)</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>(T, T)</td>
</tr>
</tbody>
</table>

Exercise: Complete the outcomes of the experiment from the previous example by tree method.

H

T

[7 - 6] Comparing between Probabilities

Example: A box contains of solid objects 7 spheres, 2 cubes and one cone.
* What is the probability drawing one sphere? More likely greater than 50%
* What is the probability drawing a cone? Less likely less than 50%
* What is the probability drawing cylinder? Impossible 0%.
* What is the probability drawing a solid object? Certain 100%

Exercise: A box contains of coloured balls, 5 green, 3 yellow and one white ball.
* What is the probability of drawing a red ball?*
* What is the percentage ratio?
* What is the probability of drawing a white ball? What is the percentage ratio?
* What is the probability of drawing any colour of ball?
* What is the percentage ratio?
Chapter Test

1. Represent the following data by frequency table.
   \[3, 12, 17, 13, 20, 18, 8, 5, 11, 16, 19, 7, 10, 15, 18, 14, 9, 14, 10, 20\]
   Which of the class has the greatest frequency.

2. A store contains sport needs, 35% sport shirts, 20% sport shoes and 45% other sport supplies, represent the data by circular sector.
   * If the store contained 200 different needs, what is the number of shoes it contained?
   * What is the value of the angle that represents the ratio 35%?

3. Represent the given data as frequency polygon and describe the changes of data.

<table>
<thead>
<tr>
<th>Class</th>
<th>3 - 7</th>
<th>8 - 12</th>
<th>13 - 17</th>
<th>18 - 22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>6</td>
<td>2</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

4. Muhammad chose socks and shoes randomly, write all the probability using tree method, if he has black and white socks, and one black shoe, brown shoes, and white shoes.

5. A box contains 3 cubes, 7 spheres and a cylindrical solid.
   * What is the probability of drawing one sphere? What is the percentage ratio?
   * What is the probability of drawing a cone solid? What is the percentage ratio?
   * What is the probability of drawing a cylindrical solid? What is the percentage ratio?
   * What is the probability of drawing a solid? What is the percentage ratio?

6. Represent the given data in (1) as stem and leaf.
   * What is the greatest and least values?
   * What is the value which has the least frequency?

7. The given table shows the sizes of sport trousers which are sailed within three weeks.
   * Represent the data as stem and leaves.
   * What is the number of sport trousers which has the size greater than 40?
   * Which of the sizes has the greatest sailed within three weeks?
   * What is the number of trousers which are sold within three weeks of size less than 40?
Choose The Correct Answer

1. The shape □ is:
   a. Concave regular.
   b. Convex hexagon regular.
   c. Concave irregular hexagon.
   d. Convex irregular hexagon.

2. The shape □ is:
   a. Convex hexagon regular.
   b. Concave regular.
   c. Convex irregular hexagon.
   d. Concave irregular hexagon.

3. Which polygon its measure of interior angles is 120°:
   a. Octagon.
   b. Pentagon.
   c. Hexagon.
   d. Quadrilateral.

4. The measure of interior angles for the decagon is:
   a. 36°.
   b. 60°.
   c. 110°.
   d. 50°.

5. Measure of exterior angle for the nonagon is:
   a. 40°.
   b. 45°.
   c. 55°.
   d. 60°.

6. Which polygon its measure of interior angles is 720°:
   a. Octagon.
   b. Pentagon.
   c. Hexagon.
   d. Quadrilateral.

7. Which polygon its measures of interior angles is 900°:
   a. Heptagon.
   b. Triangle.
   c. Pentagon.
   d. Nonagon.

8. Which polygon his measures of exterior angles is 30°:
   a. Pentagon.
   b. 12 sides.
   c. 11 sides.
   d. Decagon.

9. The shape □ is:
   a. Irregular polygon.
   b. Regular polygon.
   c. Not polygon.
   d. Convex polygon.

10. The shape □ is:
    a. Irregular polygon.
    b. Regular polygon.
    c. Not polygon.
    d. Convex polygon.

11. The shape □ is:
    a. Irregular polygon.
    b. Regular polygon.
    c. Not polygon.
    d. Convex polygon.

12. Which polygon its measure of exterior is 120°:
    a. Regular, hexagon.
    b. Regular, triangle.
    c. Regular, pentagon.
    d. Quadrilateral.
Solid Shapes and composite Solid Shapes

Choose The Correct Answer

1. What is the name of the shape △:

2. What is the name of the shape 🧐:

3. The Solids in the Picture are:

4. Which sentence is correct for the cylinder:
   a. Has one Face and one.   b. Has 3 Faces.   c. Has Bases of cylinder are circles.   d. Has 2 Vertices.

5. Which sentences are correct for the quardilateral prism:

6. What is the body which the has same distance between each point and his center, and has not vertices, bases:

7. Which body has one base and one vertex is:

8. The number of the pentagon prism’s faces are:
   a. 7 Faces.   b. 8 Faces.   c. 5 Faces.   d. 11 Faces.

9. The number of Cone’s edges are:
   a. 2.   b. 3.   c. Zero.   d. 1.

10. The number of terahedron’s vertices are:
    a. 3.   b. 1.   c. Zero.   d. 2.
Choose The Correct Answer

1. Which sentence is correct:
   a) Each X and Y coordinates for point in quadrant III are negative.
   b) Y coordinates for point on Y-axis is Zero.
   c) X-coordinate for point in quadrant I is positive.
   d) Y-coordinate for point quadrant IV is positive.

2. The coordinates of the ordered pair (3, 5) is:
   a) X = 3, Y = 5.
   b) X = 5, Y = 5.
   c) X = 5, Y = 3.
   d) X = 3, Y = 3.

3. The coordinates of the point B in the coordinate plane is:
   a) (6, 0).
   b) (0, 6).
   c) (-6, 0).
   d) (0, 3).

4. On which quadrant lies the point A on the coordinate plane?
   a) quadrant I.
   b) quadrant III.
   c) quadrant II.
   d) quadrant IV.

5. On which quadrant lies the point B on the coordinate plane?
   a) quadrant I.
   b) quadrant III.
   c) quadrant II.
   d) quadrant IV.

6. On which quadrant lies the point C on the coordinate plane?
   a) quadrant I.
   b) quadrant III.
   c) quadrant II.
   d) quadrant IV.

7. On which quadrant lies the point D on the coordinate plane?
   a) quadrant I.
   b) quadrant III.
   c) quadrant II.
   d) quadrant IV.

8. Representing the point A (5, -2) on coordinate plane we move more 5 units from the origin:
   a) Above.
   b) Right.
   c) Below.
   d) Left.

9. Representing point B (-2, 1) on coordinate plane we move more 2 units from the origin:
   a) Above.
   b) Right.
   c) Below.
   d) Left.

10. Representing point D (0, 3) on coordinate plane we move more 3 units from the origin:
    a) Above.
    b) Right.
    c) Below.
    d) Left.
Choose The Correct Answer

1. Which figures have a vertical line of symmetry:
   a.  
   b.  
   c.  
   d.  

2. Which figures have a horizontal line of symmetry:
   a.  
   b.  
   c.  
   d.  

3. How many lines of symmetry does the quadrilateral have?
   a. 5.  
   b. 4.  
   c. 3.  
   d. 2.  

4. Which figure has symmetry and which symmetry line:
   a. square, horizontal symmetry.  
   b. hexagon, horizontal symmetry.  
   c. hexagon, vertical symmetry.  
   d. hexagon, vertical symmetry.

5. Suppose the point (2, -1) is the reflection image of the point (2, 1) then the reflecting line is:
   a. X- and Y-axis.  
   b. Origin.  
   c. X-axis.  
   d. Y-axis.

6. Suppose the point (2, 3) is the reflection image of the point (-2, 3) then the reflecting line is:
   a. X- and Y-axis.  
   b. Origin.  
   c. X-axis.  
   d. Y-axis.

7. The reflection of point (5, 2) on Y-axis is:
   a. (5, 2).  
   b. (-5, 2).  
   c. (-5, -2).  
   d. (5, -2).

8. The reflection of point (0, 5) on Y-axis is:
   a. (0, 0).  
   b. (5, 5).  
   c. (0, 5).  
   d. (0, -5).

9. The reflection of triangle ABC on Y-axis is A'B'C' :
   a.  
   b.  
   c.  
   d.  

10. Translation of the point (2, 1) 2 units above and 1 unit left is the point:
    a. (1, 3).  
    b. (4, 1).  
    c. (2, 4).  
    d. (4, 2).
[ 5-5 ] Congruence and Similarity

Choose The Correct Answer

1. If the triangle ABC similar to the triangle DEF, then the length of AB is:
   a. 6.  
   b. 8.  
   c. 2.  
   d. 1.

2. The value of C in the shape is:
   a. 8.  
   b. 4.  
   c. 6.  
   d. 2.

3. Angle Measure Z in the Picture is:
   a. 20°.  
   b. 90°.  
   c. 40°.  
   d. 90°.

4. The shapes in the following pictures are:
   a. Similar.  
   b. Not Congruence.  
   c. Not Similar.  
   d. Congruence.

5. What is the shadow length for building its height is 26 m with another building its height 20 m and its shadow length is 10 m?
   a. 8.  
   b. 11.  
   c. 12.  
   d. 13.
**Rate effect on the perimeter and area**

**Choose The Correct Answer**

1. The area of a square whose side length is 9 cm under the effect of dilation coefficient 10 is equal to:
   - a) 180 cm$^2$
   - b) 810 cm$^2$
   - c) 188 cm$^2$
   - d) 288 cm$^2$

2. The perimeter of a square whose side length 6 cm under the effect of dilation coefficient $\frac{1}{2}$ is equal to:
   - a) 6 cm
   - b) 8 cm
   - c) 10 cm
   - d) 12 cm

3. The area of a circle whose center C and whose radius is 2 cm under the effect of dilation coefficient $\frac{1}{2}$ is equal to:
   - a) 153.86 cm$^2$
   - b) 76.93 cm$^2$
   - c) 177.12 cm$^2$
   - d) 254.56 cm$^2$

4. The perimeter of a circle whose center C and its radius is 50 cm under effect of dilation coefficient 5 is equal to:
   - a) 1770 cm
   - b) 1670 cm
   - c) 1570 cm
   - d) 1870 cm

5. The area of a rectangle whose sides length are 100 cm, 50 cm under the effect of dilation coefficient is equal to:
   - a) 170 cm$^2$
   - b) 180 cm$^2$
   - c) 190 cm$^2$
   - d) 200 cm$^2$

6. The perimeter of a rectangle whose sides length are 16 cm, 34 cm, under the effect of dilation coefficient 7 is equal to:
   - a) 720 cm
   - b) 700 cm
   - c) 650 cm
   - d) 610 cm

7. The area of a trapezoid whose bases 5 cm, 9 cm and height 3 cm under effect of dilation coefficient 2 is equal to:
   - a) 84 cm$^2$
   - b) 86 cm$^2$
   - c) 82 cm$^2$
   - d) 88 cm$^2$
Choose The Correct Answer

1. The volume of a cube whose edge 11 cm is equal to:
   a) 1313 cm³.  
   b) 1331 cm³.  
   c) 3311 cm³.  
   d) 1133 cm³.

2. The volume of a cube is 216 cm³, so the length of its edge:
   a) 6 cm.  
   b) 8 cm.  
   c) 10 cm.  
   d) 12 cm.

3. The parallelepiped whose dimensions are 3 cm, 6 cm so its volume is:
   a) 6 cm³.  
   b) 60 cm³.  
   c) 90 cm³.  
   d) 80 cm³.

4. The volume of a parallelepiped squared base side length 2 cm and its height three times of bases side, so its volume is:
   a) 12 cm³.  
   b) 64 cm³.  
   c) 48 cm³.  
   d) 24 cm³.

5. Height of parallelepiped whose volume 120 cm³, and the area of its base is 24 cm³ its height is equal to:
   a) 6 cm.  
   b) 5 cm.  
   c) 8 cm.  
   d) 12 cm.

6. A tank is in the form of a cube whose edge 102 m, contains oil half of its volume, the volume needed to fill is:
   a) 0.128 cm³.  
   b) 0.612 cm³.  
   c) 0.216 cm³.  
   d) 0.216 cm³.

7. A water tank in a form of parallelepiped whose dimensions 2 m, 3 m, 4 m, the water 8m/h, the time needed to fill it is:
   a) 3 hours.  
   b) 5 hours.  
   c) 8 hours.  
   d) 9 hours.
Choose The Correct Answer

1. A surface area of a cube whose edge 6 cm:
   - a) 216 cm².
   - b) 513 cm².
   - c) 128 cm².
   - d) 256 cm².

2. The area of a cube’s edge is 64 cm², its lateral length is:
   - a) 10 cm.
   - b) 8 cm.
   - c) 6 cm.
   - d) 4 cm.

3. The parallelepiped whose sides base are 5 cm, 6 cm and its height 3 cm so its area is:
   - a) 60 cm².
   - b) 66 cm².
   - c) 82 cm².
   - d) 80 cm².

4. The parallelepiped of squared base length 2 cm and its height five times of base side so the total area is:
   - a) 82 cm².
   - b) 86 cm².
   - c) 88 cm².
   - d) 84 cm².

5. A parallelepiped whose volume 120 cm³, and base area is 24 cm² so its height equals to:
   - a) 6 cm.
   - b) 5 cm.
   - c) 8 cm.
   - d) 12 cm.

6. A tank in a form of cube whose edge 2 cm, they need to paint the walls to the half its height so the painted area equals to:
   - a) 2 cm².
   - b) 4 cm².
   - c) 6 cm².
   - d) 8 cm².

7. A parallelepiped of squared whose its squared base, its height is 6 cm, and the perimeter of base is 32 cm so its total area is:
   - a) 320 cm².
   - b) 230 cm².
   - c) 160 cm².
   - d) 280 cm².
Choose The Correct Answer

1. A parallelepiped whose its dimensions are 2 cm, 8 cm, 10 cm under the effect of dilation coefficient $k = \frac{5}{2}$ is, the volume after dilation is:
   - a) 2000 cm$^3$.
   - b) 2250 cm$^3$.
   - c) 2500 cm$^3$.
   - d) 3000 cm$^3$.

2. A volume of a cube whose edge 4 cm, under the effect of dilation coefficient $K = \frac{1}{2}$ will be:
   - a) 8 cm$^3$.
   - b) 16 cm$^3$.
   - c) 24 cm$^3$.
   - d) 32 cm$^3$.

3. The total area of parallelepiped whose sides base are 10 cm, 20 cm, and its height 5 cm the total area under the effect of dilation coefficient $K = \frac{1}{5}$ will be:
   - a) 24 cm$^2$.
   - b) 18 cm$^2$.
   - c) 34 cm$^2$.
   - d) 28 cm$^2$.

4. The total area of cube whose edge 3 under effect of dilation coefficient $k = \frac{1}{5}$ is:
   - a) 36 cm$^2$.
   - b) 108 cm$^2$.
   - c) 27 cm$^2$.
   - d) 144 cm$^2$.

5. The total area of parallelepiped is 125 cm$^2$, total area under the effect of dilation coefficient is 5 cm$^2$, then the dilation coefficient is:
   - a) $\frac{2}{5}$.
   - b) $\frac{2}{5}$.
   - c) $\frac{1}{5}$.
   - d) $\frac{2}{5}$.

6. If the volume of cube is changed from 216 cm$^3$, to 8 cm$^3$, then the dilation coefficient is:
   - a) $\frac{2}{5}$.
   - b) $\frac{2}{5}$.
   - c) $\frac{1}{5}$.
   - d) $\frac{1}{3}$.

7. If the surface area of parallelepiped is changed from 72 cm$^2$ to 288 cm$^2$, then the dilation coefficient is:
   - a) 3.
   - b) 4.
   - c) 6.
   - d) 8.
### Choose The Correct Answer

1. The 3 similar cubes whose each edge length is 2 cm were put one over the anther, the volume of shape was formed is:
   
   - a) 18 cm³.  
   - b) 20 cm³.  
   - c) 22 cm³.  
   - d) 24 cm³.

2. The 3 similar cubes whose each edge length is 2 cm were put one over the anther, the total area of shape was formed is:
   
   - a) 48 cm².  
   - b) 60 cm².  
   - c) 64 cm².  
   - d) 84 cm².

3. The 5 similar cubes whose each edge length is 5 cm, 3 similar parallelepipeds whose dimensions are 1 cm, 3 cm, 5 cm, were put together, the total volume of shape was formed is:
   
   - a) 210 cm³.  
   - b) 105 cm³.  
   - c) 310 cm³.  
   - d) 215 cm³.

4. The 7 similar parallelepipeds whose dimensions are 3 cm, 3 cm, 5 cm, were put one over the anther, the volume of shape was formed is:
   
   - a) 310 cm³.  
   - b) 315 cm³.  
   - c) 320 cm³.  
   - d) 330 cm³.

5. Number of similar cubes whose each edge length 2 cm were arrange together, if the dimensions of the shape was formed are 10 cm, 20 cm, 30 cm, the number of cubes are:
   
   - a) 500.  
   - b) 600.  
   - c) 750.  
   - d) 650.

6. Number of similar parallelepipeds whose dimensions are of the shape was formed are 10 cm, 20 cm, 30 cm, the shape was formed is cube whose edge length is 90 cm, the number of parallelepipeds are:
   
   - a) 432.  
   - b) 324.  
   - c) 243.  
   - d) 243.

7. The 3 similar matches box were put one over the anther if the dimensions of bace are 2 cm, 4 cm and the height is 2 cm, then the total area is:
   
   - a) 248 cm².  
   - b) 288 cm².  
   - c) 260 cm².  
   - d) 184 cm².
Choose The Correct Answer

1. The angle of regular polygon of 7 sides is:
   a) 165.57°,  b) 182.55°,  c) 128.57°,  d) 156.55°.

2. Regular polygon whose each angle is 180°, the number of sides are:
   a) 7,  b) 5,  c) 6,  d) 9.

3. Abd-AlaH wants to pave a rectangular passage of dimensions 12 cm, 36 cm, by squared cobble, the area of one cobble is 0.5 m², the number of cobbles are:
   a) 864,  b) 648,  c) 648,  d) 846.

4. Rectangle paper whose dimensions 20 cm, 30 cm, used to pave number of regular polygon of 6 sides, the area of each one is 50 m², the number of regular polygon are:
   a) 18,  b) 16,  c) 14,  d) 12.

5. Used coins of 8 sides each one has area 24 cm², to pave the rectangular table whose dimensions 40 cm, 60 cm, the number of the coins are:
   a) 100,  b) 120,  c) 130,  d) can not the pave.

6. Used equilateral triangle cobble whose area is 2.5 m², to pave the square land whose edge length is 25 cm, the number of cobble is:
   a) 500,  b) 250,  c) 300,  d) can not the pave.

7. Can the wall be paved by 12 – sides pieces of ceramic?
   a) not possible,  b) possible,  c) possible with out condition,  d) dependent on the wall area.
1. The class 20 – 30 among is:
   - a. Every values less than 30.
   - b. Every values greater than 20.
   - c. Every values 20, 30 between.
   - d. Every values less from 20 to 30.

2. Use the frequency table for the data:
   - a. The difference between the max and min value is small.
   - b. Small.
   - c. When the data was max or the difference between max and min is large.
   - d. When the data was large.

3. The best representation for large data is:
   - a. Simple frequency table.
   - b. Signal table.
   - c. Not these.
   - d. Class frequency table with class.

4. The frequency table will help you to answer the given questions:
   - a. Some time.
   - b. Easily.
   - c. Easily and some time.
   - d. Arrange.
Choose The Correct Answer

1. The best choice to represent the percentage:
   a. Circle graph.
   b. Frequency table.
   c. Signal of counting
   d. Bar graphs.

2. The percentage 25% represents the angle sector:
   a. 120°
   b. 90°
   c. 60°
   d. 25°

3. If we have 300 trees, 30% of them orange, find the number of orange trees are:
   a. 120 trees.
   b. 90 trees.
   c. 60 trees.
   d. 30 trees.

4. If 75% the students prefer football 15% prefer and the rest prefer volleyballs, the percentage which prefer volleyballs is:
   a. 10%.
   b. 20%.
   c. 30%.
   d. 40%.

5. Which kind of data can we use in percentage:
   a. Circle graph.
   b. Stem and leaf.
   c. Frequency polygon.
   d. Bar graphs.

6. On a circle graph, 35% from 200 persons prefer the red colour, how many are they?
   a. 80.
   b. 70.
   c. 60.
   d. 5.

7. Use the pointer of the revolving dics, and choose the correct answer, when the pointer stopped on numbers which are less than 2:
   a. 25%.
   b. greater than 50%.
   c. 50%.
   d. less than 50%.
[ 7-3 ] Frequency Polygons

Choose The Correct Answer

1. The frequency polygon is a number of segments which join:
   a. Points and mid-frequency.
   b. Points and its frequency.
   c. Corner of class and center of frequency.
   d. Corner of class and its frequency class.

2. The center of class (10 – 14) in frequency polygon is:
   a. 12.
   b. 6.
   c. 8.
   d. 4.

3. Perpendicular coordinates:
   a. X-axis is frequency and Y-axis is class.
   b. X-axis is frequency and Y-axis is center of class.
   c. X-axis is center of class and Y-axis is frequency.
   d. X-axis is center of class and Y-axis is center of frequency.

4. The frequency polygon is:
   a. Closed from the right side only.
   b. Closed from the left side only.
   c. Open from both sides only.
   d. Close from both sides only.

5. The center of class (10 – 14) in frequency polygon is:
   a. 43.
   b. 33.
   c. 28.
   d. 23.
If the data consists of two grades:

a. The leaf represents ones and stem represents tens.

b. The stem represents tens.

c. The leaf represents tens.

d. The leaf represents ones.

The leaves of each stem write in the table in form:

a. Descending with repetition.

b. Ascending without repetition.

c. Ascending with repetition.

d. Descending with repetition.

The stem 3 leaves contain:

a. 39.

b. 83.

c. 38.

d. 88.

Which table can you represent by stem and leaf it?

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>669</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>669</td>
</tr>
<tr>
<td>29</td>
<td>36</td>
</tr>
<tr>
<td>6</td>
<td>53</td>
</tr>
</tbody>
</table>

Choose The Correct Answer
[ 7-5 ] Outcomes of Experiment and Organizing

Choose The Correct Answer

1. Threw two coins once, the results are:
   a) (H, T), (T, H).
   b) (H, H).
   c) (H, H), (T, T), (T, H), (H, T).
   d) (H, T).

2. Threw three coins once, the number of probabilities are:
   a) 10.
   b) 8.
   c) 6.
   d) 4.

3. We can arrange the data in from:
   a) Frequency table.
   b) Tree.
   c) Bars graph.
   d) Signal of counting.

4. The frequency polygon is:
   a) 6.
   b) 5.
   c) 4.
   d) 3.

5. Use the graph and choose the missing probability:
   a) (T, H).
   b) (H, T).
   c) (H, T).
   d) (H, H).
Comparing between Probabilities

Choose The Correct Answer

1. The probability of letter D in Baghdad word is:

2. Box contains 8 red balls, drawn one ball, the probability is:
   a. 100%.  b. Greater than 50%.  c. Less than 50%.  d. 0%.

3. Threw a dice once the probability of appearing No. 7:

4. The percentage which is the less than 50% is:

5. The pointer stops on 7:

6. The pointer stops on 5?
   a. 0%.  b. 50%.  c. Greater than 50%.  d. Less than 50%.

7. Drawing card 2 from T R L Q
   a. Less than 50%.  b. 0%.  c. 50%.  d. 100%.

8. Choose one red can?
   a. Greater than 50%.  b. 100%.  c. Less than 50%.  d. 50%. 

Drawing Z card 2 from T R L Q